MICROMORPHIC THEORY OF MULTIPHASE MIXTURES

Abstract

by

Weiming Li

We consider a general multiphase immiscible mixture whose individual components are separated by infinitesimally thin interfaces. General average balance equations for the different phases as well as for the overall mixture are derived by using a systematic spatial averaging procedure. To account for local micro-motions and micro-deformations, we model the mixture using micromorphic theory. A minimal determinate theory is obtained by taking an appropriate number of moments of the microelement balance equations for mass, momenta, angular momentum, energy, and entropy. The resulting average balance equations include equations for microinertia and microspin tensors. This model, with the phase and mixture quantities specified in a systematic manner, is shown to satisfy the 2nd law of thermodynamics and enable the modeling of immiscible multiphase materials where internal parameters, such as the volume fraction of different phases, are important. We also compare our approach with various other theories of mixtures.

We subsequently derive the balance equations of micromorphic mixtures of grade one where only first order microdeformations are considered. With the first order assumption, the number of unknown phase and mixture quantities is reduced. Specific forms of definitions for each microelement, for each phase, and for the mixture as a whole, are obtained. By applying these relations to the general
balance equations of micromorphic continuum, the balance laws of micromorphic continuum of grade one are obtained.

To demonstrate the generality of the results, we apply our theory to a two-phase model of micromorphic bubbly mixtures. The Rayleigh-Plesset equation is revisited and a generalized Rayleigh-Plesset equation is derived which include the influence of bubble inertia. The balance equations of mass, linear momentum, generalized angular momentum and energy are given. We show that the equations for microspin and microinertia, combine to yield a more general form than the Rayleigh-Plesset equation. Such an equation, in addition to accounting for the local average bubble dilatation, can also account for the local average bubble microdeformation or microstructure. Moreover, higher-order microstructural statistics which this minimal theory contains can be modeled by constitutive approximations.