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SEMINAR ANNOUNCEMENT

SPEAKER: Tariq Aslam, Visiting Scholar
Department of Aerospace and Mechanical Engineering
University of Notre Dame, Notre Dame, Indiana

TOPIC: From Accurate Hyperbolic Partial Differential Equation Solvers To Parallel Parking Your Car and Avoiding Traffic Tickets: Applications of Level Set Methods

DATE: Tuesday, September 22, 2009

TIME: 3:30 p.m.

PLACE: 138 DeBartolo Hall

ABSTRACT

Level set methods [1] are a mathematical/computational tool for representing an interface (front). In particular, a front of dimension N-1 may be represented by a isosurface of an N dimensional function. A simple example would be representing a circle (a 1 dimensional surface) by a contour of a cone (a 2 dimensional function). So, the circle is thought of as a particular "level set" of the cone function. Level set methods have been applied to a wide variety of computational physics problems ranging from optimal path planning in robotic navigation to fluid mechanics, grid generation and combustion [2,3].

Here, a level set algorithm for tracking discontinuities in hyperbolic conservation laws is presented [4,5]. The algorithm uses a simple finite difference approach, analogous to the method of lines scheme presented in [6]. The zero of a level set function is used to specify the location of the discontinuity(ies). Since a level set function is used to describe the front location, no extra data structures are needed to keep track of the location of the discontinuity. Also, two solution states are used at all computational nodes, one corresponding to the "real" state, and one corresponding to a "ghost node" state, analogous to the "Ghost Fluid Method" of [7]. High-order, point-wise convergence is demonstrated for linear and nonlinear scalar conservation laws, even at discontinuities and in multiple dimensions. The solutions are compared to standard high-order shock-capturing schemes.

This presentation will also present work on systems of conservation laws. In particular, results for the multi-dimensional Euler equations will be presented. Examples will include tracking of material interfaces and tracking of shock waves. Applications ranging from compressible fluid instability problems (Rayleigh-Taylor, Richtmyer-Meshkov and Kelvin-Helmholz) to tracking shocks in reactive fluid flow will be presented. It will be demonstrated that the method can be used effectively when very accurate results are required for problems involving discontinuities.

Finally, some interesting applications of the level set methods for optimization, control and path planning will be presented.

[1] S. Osher and J. Sethian, "Fronts propagating with Curvature-Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulation," Journal of Computational Physics, 79, pp 12-49, 1988

[2] J. Sethian, "Level Set Methods and Fast Marching Methods: Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Material Science," Cambridge University Press, 2nd Ed., 1999

[3] S. Osher and R. Fedkiw, "Level Set Methods and Dynamic Implicit Surfaces," Springer-Verlag, 2003

[4] T.D. Aslam, "A Level Set Algorithm for Tracking Discontinuities in Hyperbolic Conservation Laws I: Scalar Equations," Journal of Computational Physics, 167, pp 413-438, 2001

[5] T.D. Aslam, "A Level Set Algorithm for Tracking Discontinuities in Hyperbolic Conservation Laws II: Systems of Equations," Journal of Scientific Computing, 19, pp 37-62, 2003

[6] C.-W. Shu and S. Osher, "Efficient Implementation of Essentially Non-oscillatory Shock-Capturing Schemes," Journal of Computational Physics, 77, pp 439-471, 1988

[7] R. P. Fedkiw, T. Aslam, B. Merriman and S. Osher, "A Non-Oscillatory Eulerian Approach to Interfaces in Multimaterial Flows (The Ghost Fluid Method)," Journal of Computational Physics, 152, pp 457-492, 1999