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INFORMAL COFFEE PERIOD BEFORE THE SEMINAR IN ROOM 365, ENGR. BLDG.

UNIVERSITY OF NOTRE DAME, NOTRE DAME, INDIANA 46556

SPEAKER: Daniel A. Tortorelli (BSME ND 1984)
Department of Mechanical Science and Engineering
University of Illinois at Urbana-Champaign
Urbana, Illinois

TOPIC: **STRUCTURAL TOPOLOGY OPTIMIZATION**

DATE: Thursday, April 8, 2010

TIME: 3:30 p.m.

PLACE: 118 Nieuwland Science

ABSTRACT

Since the popular 1988 work of Bendsøe and Kikuchi, structural topology optimization has been a thriving research topic. In this talk I discuss the history of and three resolutions to the problem:

$$\begin{array}{ll} \min\limits_{f \in O} & c_\Omega(u) \\ \text{such that} & a_\Omega(u,v) = l_\Omega(v) \, \forall \, v \in H_0^1 \end{array}$$

where a_Ω and l_Ω are the energy bilinear and load linear forms associated with the equations of linear elasticity, u and v are test and trial displacement fields, c_Ω is the cost function to be minimized, e.g. the compliance $c_\Omega = l_\Omega$, f —the control to be optimized—defines the region Ω over which c_Ω, a_Ω and l_Ω are defined and O is the set of admissible controls which provides smoothness and volume restrictions on Ω.

The genesis of topology optimization is attributed to the inability to generate holes in structural shape optimization wherein it is common to express Ω as the image of a fixed region $\hat{\Omega}$ so that $O = \{f(\hat{\Omega}) : f \text{ is a diffeomorphism}\}$, e.g. f can be a mapped mesh generator. The diffeomorphism restriction on the control f precludes the ability to introduce holes into the structure. This limitation is resolved in the *topological derivative* approach wherein an asymptotic analysis is used to determine the effect that a small hole of size ϵ placed at the location $x \in \Omega$ has on the value of the cost function c_Ω.

Another way to introduce holes is to define the set of admissible controls as $O = \{\chi : \chi(x) = 0 \text{ or } 1 \text{ for all } x \in \Omega_o\}$ where χ is the characteristic function of Ω and Ω_o is the fixed hold-all. This leads to an ill-posed problem which can be physically justified by noting that a series of many strategically placed fine holes renders a stiffer structure than a series of fewer strategically placed large holes of equal net volume. The limit in this refinement procedure, and hence the solution to the topology optimization problem does not exist. To make the problem well-posed the admissible control set O is changed in one of two ways. In the *relaxation method* generalized designs are introduced via perforated composite materials with infinitesimal holes so that $O = \{\rho : 0 \leq \rho(x) \leq 1 \text{ for all } x \in \Omega_o\}$ where the control ρ is the solid material volume fraction. This approach requires homogenization to define the elasticity tensor and the allusive $\mathcal{G}_\rho$-closure to quantify the range of elasticity tensors for a given volume fraction $\rho(x)$. In the *restriction method* constraints are placed on χ to provide a minimum length scale for the geometric features of Ω, e.g. by limiting the perimeter.

The aforementioned methods are amenable to computation and we provide the following linear elastic examples to illustrate them:

1. To compare the methods, the classical problem of minimizing the compliance of a cantilever beam subject to a volume constraint is solved via all three methods,
2. To illustrate the ability to accommodate localized constraints, the minimal weight design of a bracket subject to stress constraints is solved via restriction,
3. To demonstrate the ability to accommodate dynamics, maximal energy focus and dispersion designs of two-phase composite plates subject to impact loads are solved via relaxation, and
4. To motivate other applications of the topological derivative, energy release rates are estimated for cracks of *any* small size, at *any* boundary location and at *any* orientation from a *single* finite element analysis on a *coarse* mesh.

NOTE: ***If you are interested in meeting individually with Prof. Tortorelli, please contact Evelyn at 631-5431***