STABILITY OF COUPLED REACTORS

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Colleen F. Reidy

Dr. Mihir Sen , Director

Department of Aerospace and Mechanical Engineering

Notre Dame, Indiana

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ABSTRACT

This thesis examines whether or not coupling a system of chemical reactors will affect the overall stability of the system. First, the heat transfer involved in a single reactor is analyzed, and three different steady state solutions are found. Depending on the convection and heat generation constants involved in the system, there are two possibilities for a stable solution and one possibility for an unstable solution.

This analysis is then carried forward into a system of coupled reactors, set up in a ring so as to neglect the end conditions and focus on the interaction between the reactors. Because of this setup, there is an additional component of conduction transferring heat between the reactors, which is dependent on the temperatures of the other reactors. The heat equation for a single reactor becomes a circulant matrix with known eigenvalues that dictate the stability of the system. It is determined that even with the added conduction , the situations that would have been stable for a single reactor remain stable for a coupled reactor system, but the unstable situations for a single reactor remain unstable for coupled reactors.

Therefore, it is concluded that there is no difference in stability between a single reactor and coupled reactors. However, to avoid the unstable steady state solution, the parameters of the system must be adjusted so that the convection is greater than the heat generation. This can be accomplished by increasing the surface area of the reactors that is exposed to air or fluid, or increasing the convection coefficient by using a fluid instead of air or adding a fan.

CONTENTS

FIGURES

TABLES

SYMBOLS

A parameter governing linear stability, $f'(\overline{T})$

A_s surface area

c constant

c_v specific heat capacity at a constant volume

E activation energy of the reaction

$f(T)$ heat equation for reactor

$f(\overline{T})$ heat equation for steady state

h convection coefficient of reactor

j count variable for coupled reactors

K conduction coefficient between reactors

k rate of the reaction.

m mass of reactor

R universal gas constant

T temperature of reactor, dimensionless

T_i temperature of ith reactor, dimensionless

T^* temperature of reactor, dimensional

T_∞ temperature of surroundings, dimensionless

T_∞^* temperature of surroundings, dimensional

$\dot{T}$ change in temperature with respect to time, dimensionless

$\overline{T}$ steady state solution of T, dimensionless

$T(t)$ small perturbation of T with respect to time

$T'(t)$ change in T with respect to time

t time, dimensionless

t^* time, dimensional

β parameter equivalent to hAE/cR, dimensionless

$\beta_{c,1}$ upper critical value of β

$\beta_{c,2}$ lower critical value of β

CHAPTER 1

INTRODUCTION

Reactors, whether single or coupled, are used in a variety of industries for many purposes. From batteries to gas-to-liquid reactors, the function of these reactors is crucial to everyday life. The examples named are commonly used as sources of power. Coupled reactors, or reactors in close proximity to each other, are often used not only to save space, but also to save energy. An exothermic reaction can provide the heat to an endothermic reaction, thus minimizing the need to provide additional heat [1]. Because heat is involved in the function of these reactors, there is concern that the rate at which the temperature of the reactor is changing may actually cause the reactor to become unstable and experience thermal runaway. Thermal runaway occurs when the reaction spurs itself on, so to speak. If the rate of the reaction is dependent on temperature and there is a significant increase in temperature, the rate of the reaction will increase. This thereby increases the temperature, which increase the reaction rate, and so on until the reaction goes out of control, or "runs away." If this is possible with a single reactor, it logically seems to follow that when the reactors are coupled together, the additional heat transfer between the reactors will cause the system to become less stable.

In the case of batteries, thermal runaway will occur in when a certain voltage threshold is reached. This particular thermal runaway is due to the irreversible heat transfer occurring within the battery [2]. Once a certain critical voltage is reached, the reactor will become unstable [3]. The instability can therefore cause fires and explosions, especially in the case of lithium ion batteries [4].

When the reactors are coupled together, there is an additional component of heat transfer affect the temperature and therefore reaction rate of the reactor. If one reactor in a set becomes unstable, it will likely cause the other reactors to become unstable due to the heat transfer between the reactors. One of the proposed solution to this risk is to monitor each reactor separately for thermal runaway [4].

The following details an investigation as to whether or not coupling the reactors will make the system less stable. First, the parameters dictating the stability of a single reactors are found, which then leads into studying the parameters of coupled reactors. The coupled reactors will be modeled in a ring, similar to previous research [5]. The ring allows the end conditions, which are unknown in this case, to be neglected. The method is then verified by using a numerical method to studying the relationship of the reactor temperature over a period of time with varying parameters. The resulting analysis will show whether or not coupling the reactors will weaken the stability of the system, under what conditions instability could occur, and how to adjust the conditions to maintain the stability of the system.

CHAPTER 2

SINGLE REACTOR

2.1 Motivation

In order to understand the potential instability in coupled reactors, the potential instability of a single reactor must be understood. The following analysis considers the heat transfer involved in a single reactor- the heat generated from the internal chemical reaction and the heat transfer due to convection.

2.2 Chemical Reaction

The reactors in question are powered using a chemical reaction. The Arrhenius rate equation models how properties of a chemical reaction will change as temperature changes. For a single reactor, the equation modeling the exothermic reaction in relation to temperature was used:

$$k \approx e^{-E/RT}, \tag{2.1}$$

where k is the rate of the reaction, E is the activation energy of the reaction, R is the universal gas constant, and T is the temperature. The product RT also represents the average kinetic energy of the system, making the exponent the ratio between the activation energy and the average kinetic energy of the molecules. When the molecules have a greater kinetic energy than activation energy, there are more collisions occurring and thus more molecules reacting. Therefore, the higher the temperature,

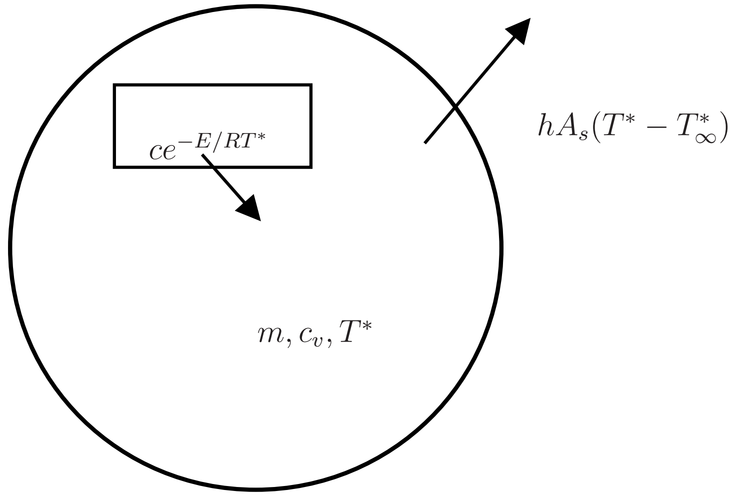

Figure 2.1. Schematic of the heat transfer of a single reactor.

the faster the reaction [6]. By itself, this equation also models the potential thermal runaway of the system.

2.3 Heat Equation

In addition to the heat generated from the reaction, there is an additional convective heat transfer from the surface of the reactor to the surroundings (Fig. 2.1). Combining the heat generated within the reactor and the heat leaving the reactor by convection, a relationship can be derived between those and the change in temperature with respect to time (Eq. 2.2). This is more commonly known as the heat equation:

$$mc_v \frac{dT^*}{dt^*} = ce^{-E/RT^*} - hA_s(T^* - T^*_\infty), \tag{2.2}$$

where t^* is time, T^* is the temperature of the system, T^*_∞ is temperature of the surroundings, m is the mass of the reactor, c_v is the specific heat capacity at a constant volume, c is a constant scale for the Arrhenius rate equation, h is the convection coefficient, and A_s is the reactor surface area. For this particular equation, all of these variables are dimensional.

2.4 Non-dimensionalization

In order for this relationship to be used for multiple scales or applications, the dimensions of Eq. 2.2 must be removed. Taking

$$T^* = \frac{E}{R}T, t^* = \frac{mc_vE}{cR}t, \tag{2.3}$$

the dimensions of the constant parameters cancel out, making T and t nondimensional.

Substituting T and t for T^* and t^*, respectively, into Eq. 2.2 gives [7]

$$\dot{T} = e^{-1/T} - \beta(T - T_\infty), \tag{2.4}$$

$$= f(T), \tag{2.5}$$

where $\dot{T}$ represents the derivative of T with respect to time (t) and there is one parameter $\beta = (hA_sE/cR)$. β is the relationship between all of the constants in the heat equation (Eq. 2.2) and is therefore dependent on the properties of the system. It is also worth noting that the greater the convection coefficient or the surface area, the greater the value of β. For simplicity, the right-hand side of Eq. 2.4 will be referred to as $f(T)$ (Eq. 2.5) throughout this paper.

2.5 Chosen Parameters

For Eq. 2.5, the steady state temperature occurs when the temperature is no longer changing. In other words, the steady-state solution is when

$$f(\overline{T}) = 0, \tag{2.6}$$

where $\overline{T}$ is the steady state solution.

For different values of β, there are three possibilities for $\overline{T}$, as shown in Fig. 2.2.

(a) One (an upper) solution exists for $\beta < \beta_{c,1}$.

(b) Three (an upper, an intermediate, and a lower) solutions exist for $\beta_{c,1} < \beta < \beta_{c,2}$.

(c) One (a lower) solution exists for $\beta > \beta_{c,2}$.

In this case, the critical values $\beta_{c,1}$ and $\beta_{c,2}$ are the β values where the number of $\overline{T}$ values (or steady-state solutions) changes from one to three or from three to one, respectively, for increasing values of β. These critical values depend on βT_∞, which are the constant parameters of the system. For this entire analysis, T_∞ was taken to be 0.18, which allowed for consistency and clean graphs. The β values shown in the figure are the β values that best represented the three different possibilities for $\overline{T}$.

In determining the values for $\beta_{c,1}$ and $\beta_{c,2}$, the points where the function $f(\overline{T}) = 0$ were calculated for various values of β by plotting Eq. 2.5 for various values of β and finding the steady state solutions. (Table 2.1).

From these calculations, the value for $\beta_{c,1}$ was determined to be 0.259 and the value for $\beta_{c,2}$ was 0.462.

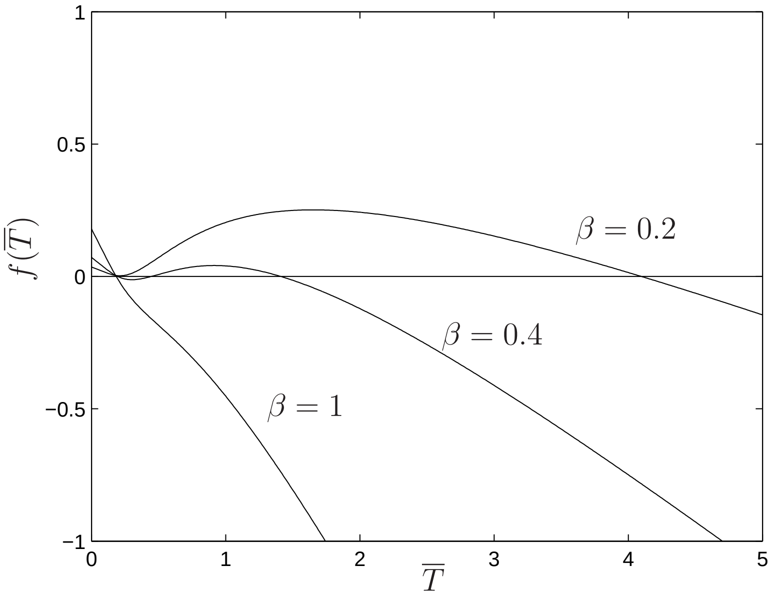

Figure 2.2. $f(\overline{T})$ with $T_\infty = 0.18$ and different βs

2.6 Stability

The steady state solutions previously calculated may or may not be stable. Using the values from Table 2.1, the steady state solutions ($\overline{T}$) were plotted a function of β. As shown in Fig. 2.3, for small values of β the single $\overline{T}$ value where $f(\overline{T}) = 0$ was relatively high, for middle values of β there were three $\overline{T}$ values, and for large values of β there was a single, low value of $\overline{T}$.

Because Eq. 2.5 is a nonlinear differential equation (due to the exponential), it cannot be solved to determine the linear stability of the steady states. Therefore, the equation was modeled as linear over small changes, or perturbations. A small perturbation $T(t)$ was given such that

$$T(t) = \overline{T} + T'(t). \tag{2.7}$$

TABLE 2.1

Calculating the critical values of β

β *Value*	T_1	T_2	T_3	
0.2			4.0972	
0.25			3.0671	
0.259	0.229	0.2427	2.922	$\beta_{c,1}$
0.3	0.206	0.3033	2.3633	
0.35	0.1986	0.3662	1.8385	
0.4	0.1947	0.4473	1.4102	
0.45	0.1922	0.6017	0.9859	
0.462	0.1918	0.6859	0.8548	$\beta_{c,2}$
0.5	0.1905			
0.6	0.1882			
0.8	0.1857			
1	0.1844			

In other words, $T(t)$ represents the temperature when a small change $T'(t)$ has been applied to the steady state value. Linear stability of $\overline{T}$ is then governed by

$$\dot{T}' = AT', \tag{2.8}$$

where

$$A = f'(\overline{T}). \tag{2.9}$$

When A is negative, high values of T will be driven back to the steady state value. Therefore, the steady states with $f'(\overline{T}) < 0$ are stable. However, as shown in Fig. 2.2,

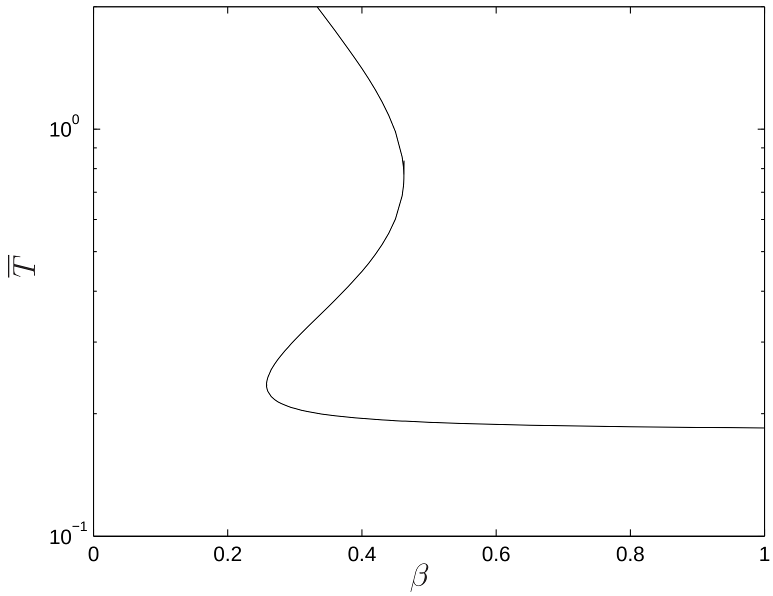

Figure 2.3. $\overline{T}$ values when $f(\overline{T}) = 0$ for different βs

for certain temperatures $f'(\overline{T}) > 0$ for the intermediate values of β. Therefore, when the value of β is between the two critical values (as found in Table 2.1), the solution will be unstable. Thus, when the steady state solutions exist, the lower and upper solutions will be stable, but the intermediate will be unstable. Additionally, since these solutions are dependent on β, the solutions are dependent on the parameters of the system, namely the constants involved in convection and in the heat generation.

CHAPTER 3

COUPLED REACTORS

3.1 Motivation

The stability of the single reactor has been analyzed. A single reactor can have a period of instability dependending on the properties and parameters of the system. The single reactor in this analysis, however, is only dependent on the internal chemical reaction and the convection to the surroundings. When the reactors are coupled, there is an additional component of conduction between the reactors. The following analysis examines how the conduction could affect stability.

3.2 Governing Equations

Just as with the single reactor, there is a component of internal heat generation. Again, this is modeled using the Arrhenius rate equation

$$k \approx e^{-E/RT_i}, \tag{3.1}$$

where T_i is the temperature of the reactor in question. This is exactly the same as for a single reactor. Likewise, there is also a component of convection to the surroundings. Using the same analysis as for a single reactor (i.e. non-dimensionalization of the heat equations), this relationship is found:

$$\dot{T}_i = e^{-1/T_i} - \beta(T_i - T_\infty) \tag{3.2}$$

$$= f(T_i). \tag{3.3}$$

Again, T_i is the temperature of the reactor in question. This relationship is exactly the same as the one for the single reactor.

However, there will also be a conduction component between the reactors. Since the reactors are arranged in a ring to neglect the end conditions, the conduction will occur between the reactor in question and the reactors on either side. Combining the conduction between the reactors with the heat transfer from the single reactor gives this heat equation:

$$\dot{T}_i = f(T_i) + K\left(T_{i-1} - 2T_i + T_{i+1}\right), \tag{3.4}$$

where K represents the conduction coefficient between each reactor (and therefore $K > 0$), i represents the reactor in question, and the parameter β is assumed to be the same for all of the reactors.

3.3 Stability

Similar to the single reactor, steady state solutions satisfy

$$f(\overline{T}_i) + K\left(\overline{T}_{i-1} - 2\overline{T}_i + \overline{T}_{i+1}\right) = 0, \tag{3.5}$$

or when the temperature of the reactor is no longer changing.

Because of the exponential function, Eq. 3.5 is a transcendental equation, and therefore does not always have a finite set of roots. Transcendental equations can have no roots, a finite set of roots, or an infinite set of roots. These roots can either be real or complex. This analysis only investigated real roots because the complex roots would add a component of oscillation that was beyond the scope of this study. The system was studied for the possibilities of a single root (i.e. all reactors are the same temperature) and two roots (i.e. alternating the reactor temperatures around the ring).

3.3.1 Single Root

Having a single root for Eq. 3.5 can be written as $\overline{T}_i = \overline{T}$ where $f(\overline{T}) = 0$. The linear stability of this is governed by

$$\dot{\mathbf{T}}' = \mathbf{A}\mathbf{T}', \tag{3.6}$$

where $\mathbf{T}'$ is a vector of perturbations for each reactor

$$\mathbf{T}' = \begin{bmatrix} T_1' \\ T_2' \\ \vdots \\ T_n' \end{bmatrix}, \tag{3.7}$$

and $\mathbf{A}$ is a circulant matrix of the form

$$\mathbf{A} = \begin{bmatrix} f'(\overline{T}) - 2K & K & \dots & 0 & K \\ K & f'(\overline{T}) - 2K & K & \dots & 0 \\ \vdots & & & & \\ K & 0 & \dots & K & f'(\overline{T}) - 2K \end{bmatrix}. \tag{3.8}$$

Circulant matrices are matrices where the terms seem to rotate around each other. For example, the $4x4$ matrix

$$\mathbf{M} = \begin{bmatrix} c_0 & c_3 & c_2 & c_1 \\ c_1 & c_0 & c_3 & c_2 \\ c_2 & c_1 & c_0 & c_3 \\ c_3 & c_2 & c_1 & c_0 \end{bmatrix} \tag{3.9}$$

is circulant because each term moves a place to the right and the rightmost terms move to the leftmost position as they go down the rows. Eigenvalues for an nxn circulant matrix are given by

$$\lambda_j = c_0 + c_1\omega_j + c_2\omega_j^2 + c_3\omega_j^3 + \cdots + c_{n-1}\omega_j^{n-1}, \tag{3.10}$$

where

$$\omega_j = e^{(2\pi ij/n)}, \tag{3.11}$$

and $j = 0 \ldots n$-1 and $i = \sqrt{-1}$. The value of ω_j will travel along the unit circle, therefore causing the eigenvalues to oscillate between -1 and 1 [8].

Therefore, the n eigenvalues of the symmetric, circulant matrix $\mathbf{A}$ from Eq. 3.8 are [9]

$$\lambda_j = f'(\overline{T}) - K\left(2 - e^{2\pi ji/n} - e^{2\pi ji(n-1)/n}\right) \tag{3.12a}$$

$$= f'(\overline{T}) - 2K\left(1 - \cos\frac{2\pi j}{n}\right), \tag{3.12b}$$

for $j = 0, 1, \ldots, n-1$. If all $\lambda_j < 0$, then the system will be stable. Since $f'(\overline{T})$ can be positive or negative depending on β, Eq. 3.12b will need to be evaluated for both situations. Each solution reacts differently: (i) If $f'(\overline{T}) < 0$, i.e. for the lower and upper solutions, the previously stable solutions continue to be stable. This is because K is always greater than 0, and the term next to it can never be negative because cosine oscillates between -1 and 1. Therefore, if the $f'(\overline{T})$ term is negative, the K term will always be subtracted from it, therefore making the eigenvalues more negative. Thus, the system remains stable. (ii) If $f'(\overline{T}) > 0$, i.e. for the intermediate solution, the previously unstable solution may seemingly be stable if

$$K > \frac{f'(\overline{T})}{2\left(1 - \cos 2\pi j/n\right)}, \tag{3.13}$$

for all $j = 0, 1, \ldots, n$. However, when $j = 0$, the denominator becomes 0 and therefore the minimum required value of K approaches infinity. If $\lambda_0 > 0$, not all λ_j are negative, and therefore the system will be unstable. Therefore, for a single root (or fixed point) of Eq. 3.5, the systems that cause a single reactor to be stable will continue to be stable and the systems that cause a single reactor to be unstable will continue to be unstable.

3.3.2 Two Roots

Another solution to Eq. 3.5 that was studied was the possibility that every other reactor would have the same temperature. In other words, the even reactors would have one temperature and the odd reactors would have another temperature. The methodology used was the same as the single root, and the equations were solved numerically. The results are in the next section.

CHAPTER 4

NUMERICAL RESULTS

4.1 Motivation

To understand how the temperature of the reactors changes over time, the numerical solutions to Eq. 2.4 and Eq. 3.4 were calculated using Newton's Method. For each plot, the steady state solutions are plotted as dashed lines.

4.2 Single Reactor

For a single reactor, the approximate solution for Eq. 2.4 was calculated and plotted for different initial values of T, as shown in the following figures.

Fig. 4.1 shows how the temperature of the reactor changes over time from various initial conditions when $\beta = 0.2$. The steady state solution (the dashed line), because it is at a high temperature, can be interpreted as the ignition point . No matter what the initial condition of the reactor is, it will eventually reach the steady state and simply burn. Because each of the initial conditions approaches the steady solution, this is stable.

Fig. 4.2 shows how various initial conditions for the temperature change over time when $\beta = 0.4$. There are three steady-state solutions- the top dashed line can be considered the ignition point (i.e. when the reactor catches on fire). If the reactor starts at a temperature higher than this steady state, it will eventually cool down to the steady state temperature while still burning. The bottom steady state can be considered the normal running temperature of the reactor. As long as the initial

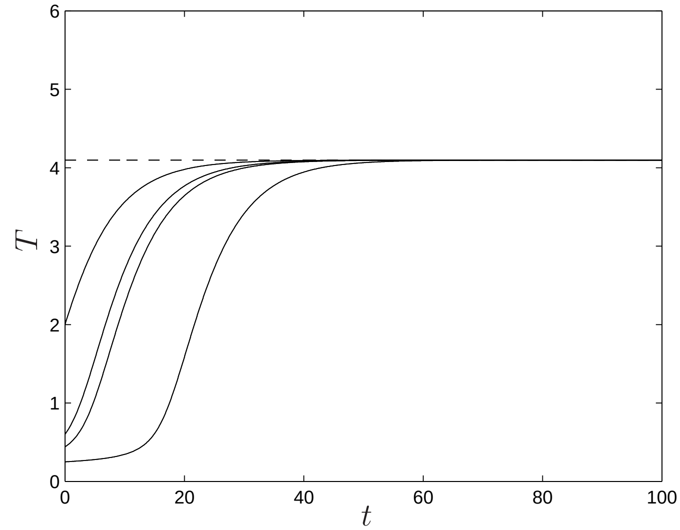

Figure 4.1. T of a single reactor for various initial values of T, where $\beta = 0.2$

condition is below the middle steady state value, the reactor will cool to the lowest steady state. However, the middle steady state is unstable in that all of the initial conditions tend away from that steady state. If the initial condition is above the middle steady state value, it will quickly tend to the high steady state, which is the ignition point. In other words, the reactor will catch on fire. Therefore, this entire system can be considered to have two attractors. Any initial value of T will tend towards either one of these two attractors: the normal operating temperature of the reactors or the ignition point.

Fig. 4.3 shows how the temperature of a reactor changes over time at different initial conditions when $\beta = 1$. Just as when $\beta = 0.2$, there is one steady state solution. Because the steady state solution (the dashed line) is at a low temperature, this can be taken as the standard operating temperature of the reactor. Each of the initial conditions approaches the steady state solution and will remain there, not heating up or igniting.

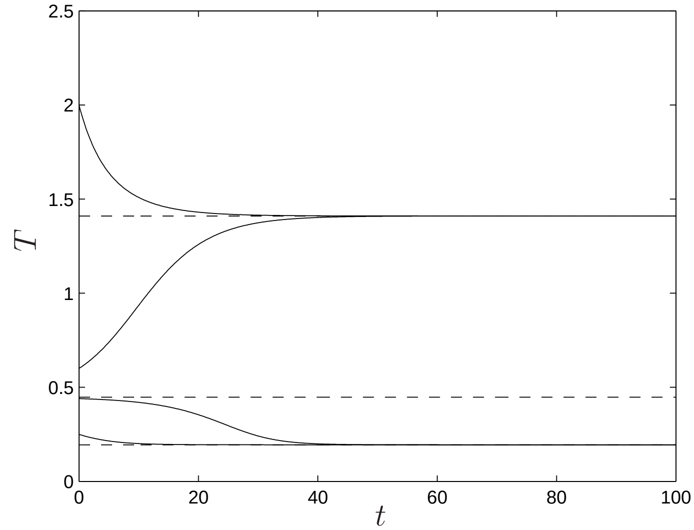

Figure 4.2. T of a single reactor for various initial values of T, where $\beta = 0.4$

The numerical results for the single reactor prove what was previously determined. For values of β between the two critical values, which is a constant reflecting the parameters of the reactor system, there is an unstable solution. If the initial temperature is above the middle steady state solution, the reactor will heat up quickly, thus resulting in thermal runaway.

4.3 Coupled Reactors

For a coupled reactor system of 10 reactors, the even numbered reactors were presumed to have an initial condition higher than the odd numbered reactors. Eq. 3.4 was then solved using Newton's method to determine the temperature of two of the reactors, one even and one odd.

Fig. 4.4 shows the change in temperature over time for the coupled reactors when $\beta = 0.2$. Just as with the single reactor, the steady state solution (the dashed line) is at a high value and therefore can be presumed to be the ignition point for

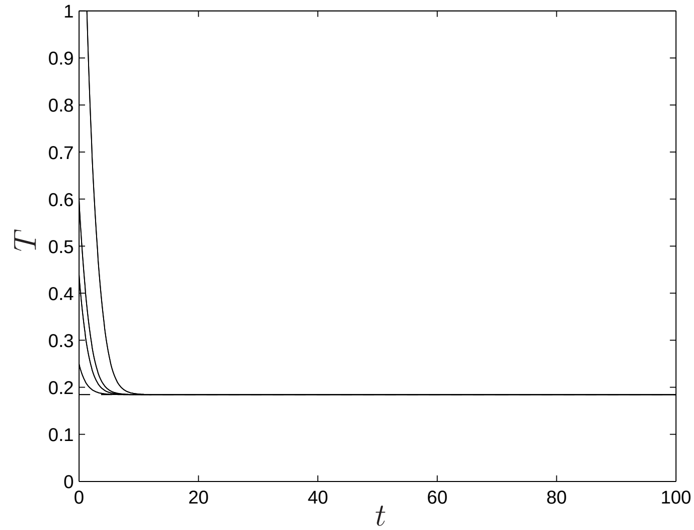

Figure 4.3. T of a single reactor for various initial values of T, where $\beta = 1$

the reactors. Both the even and odd reactors approach the steady state solution, meaning that they will both heat up until they are burning. Because all of the initial conditions approach this steady state value, this is stable.

Fig. 4.5 shows the temperature over time for the coupled reactors when $\beta = 0.4$. The even reactors have an initial temperature above the ignition steady state temperature (the dashed lines) and the odd reactors have an initial condition below the middle steady state temperature. Just as with the single reactor, the even reactors cool to the ignition point and continue to burn, whereas the odd reactors cool to the normal operating temperature and remain there. It seems that the convection of the system is significant enough to dissipate the heat given off by the even reactors and therefore preventing the odd reactors from heating up as well. This situation requires further investigation.

Fig. 4.6 shows the temperature as a function of time for the coupled reactors when $\beta = 1$. Just as with the single reactor, both the even and odd reactors tend toward the low steady state temperature (the dashed line), which can be taken as

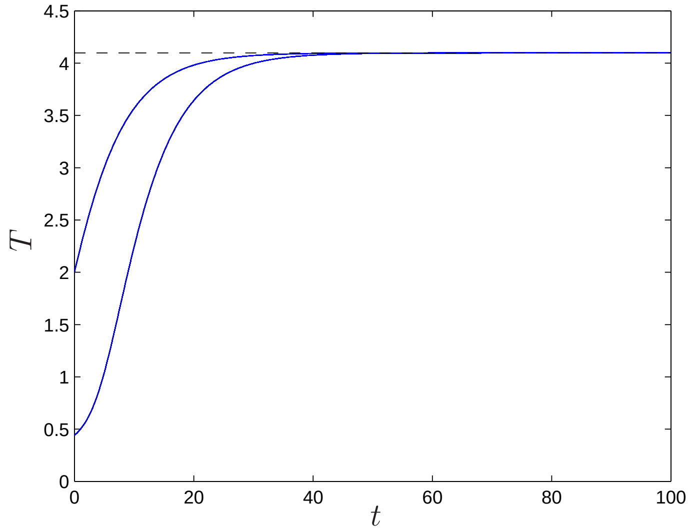

Figure 4.4. T of coupled reactors for 2 initial conditions with $\beta = 0.2$.

the normal operating temperature for the system. Again, because all of the initial conditions approach the steady state value, this is considered stable.

As a point of comparison, the plot of the coupled reactors when $\beta = 0.4$ (blue) was plotted over the plot of the single reactor when $\beta = 0.4$ (black) (Fig. 4.7). Because the coupled reactor plot perfectly overlays the single reactor plot, it can be concluded that there is no change in the stability of the reactors between a single reactor and a system of coupled reactors.

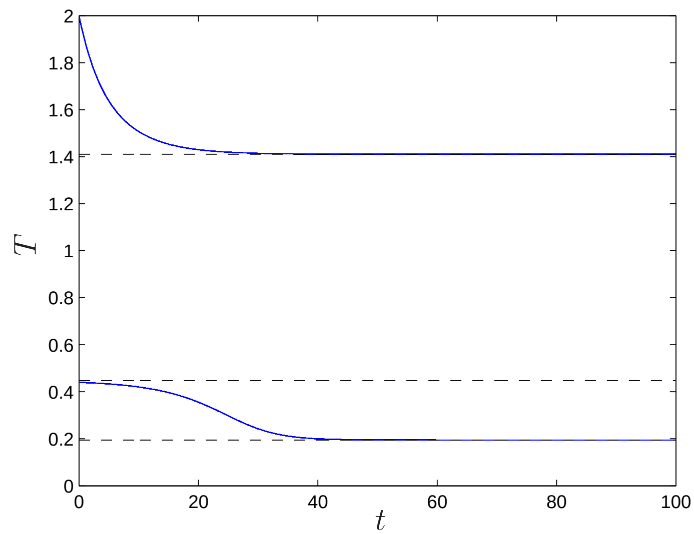

Figure 4.5. T of coupled reactors for 2 initial conditions with $\beta = 0.4$.

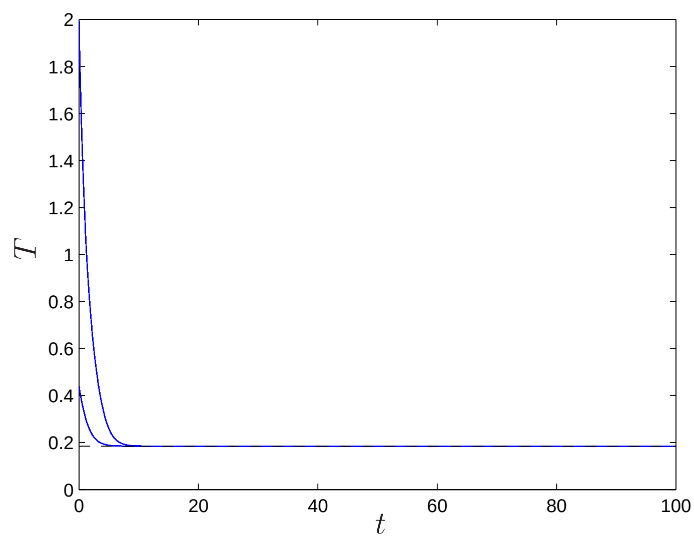

Figure 4.6. T of coupled reactors for 2 initial conditions with $\beta = 1$

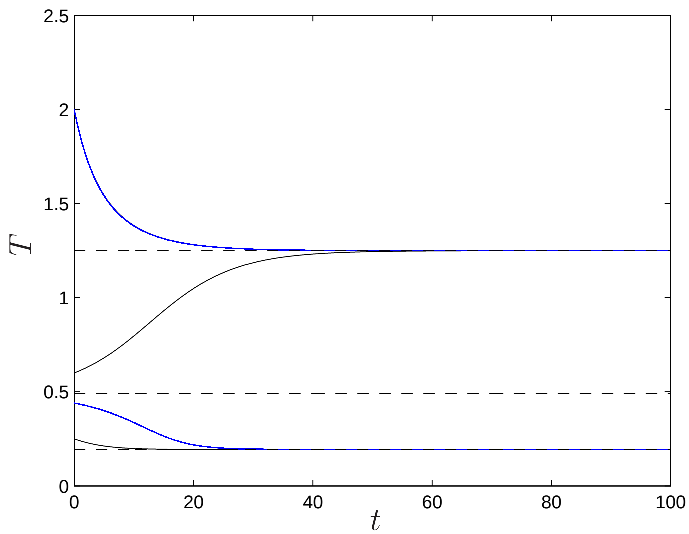

Figure 4.7. T of coupled reactors over T of a single reactor for various initial values of T, where $\beta = 0.4$

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Summary

This paper investigated the potential instability of coupled reactors in order to determine whether or not placing the reactors in close proximity to each other makes the system more or less stable. The system was analyzed in a ring so as to neglect the unknown end conditions. First, the stability of a single reactor was investigated by analyzing the heat transfer involved. It was discovered that depending on the parameters of the system, there were three potential solutions: a stable solution where the reactors approached at a low operating temperature, an unstable solution where the reactors would quickly ignite, and a third where the reactors would approach a steady high temperature, where the reactors could be considered to have ignited.

This analysis was then applied to the coupled reactor system. Each individual reactor had the same heat transfer as the single reactor system, but with an additional component of conduction between the reactors. Two situations were examined, one where the temperature of each reactor was at the steady state temperature (i.e. where the temperature was no longer changing), and one where the initial temperature of the even numbered reactors was higher than the initial temperature of the odd numbered reactors. It was found that the situations where a single reactor would be unstable were also the situations where coupled reactors would be unstable, and likewise when the single reactor would be stable, the coupled reactors would be stable. Therefore, it was concluded that coupling the reactors did not affect the stability of the reactors, at least for those two situations.

5.2 Applications

The analysis performed can be used in industry to determine whether or not to use single or coupled reactors and how the stability of the system will be affected. As mentioned above, it was determined that whether or not the reactors are coupled has very little to do with the stability of the system. The stability has far more dependence on the parameters of the system than whether or not the reactors are coupled. The primary parameter in the analysis was the relationship between the convection and the heat generation constants involved in the heat transfer of the reactor. As mentioned before, the stable solutions were those where the reactor was either approaching a low operating temperature or approaching a high ignition temperature. For most industrial applications, fire tends to be avoided and therefore the system that approaches a low operating temperature would be best. This solution correlates to the convection constants being greater than the heat generation constants. To prevent a system becoming unstable, therefore, one can increase the surface area of the system or adjust the convection coefficient by adding a fan or a fluid to increase the heat transferred through convection. This way, whether the reactors are single or coupled, they should remain stable.

5.3 Limitations and Recommendations

This analysis still depends on several assumptions. First, the coupled reactors were placed in a ring to neglect the end conditions. In industrial settings, placing the reactors in a ring may not be possible, and therefore the end conditions could affect the behavior of the reactors. Because this analysis focused on the interaction between the reactors, the end conditions could be ignored. Additional studies could look into the possible effects of including end conditions.

Additionally, the chemical reaction within the reactors was modeled using a constant before the Arrhenius rate equation. As the reaction proceeds within the reactor, the proportion of reacted molecules to unreacted molecules will change, which could have an effect on the significance of the chemical reaction. In other words, as the fuel, so to speak, is used, there will be less fuel to proceed with the rest of the reaction, which could therefore change the influence of the Arrhenius equation (i.e. change the constant in front of the equation). This could affect the stability of the reactors when the system is analyzed for a long period of time. Further studies could look into the effects of this phenomenon.

Lastly, in the analysis of the coupled reactors, only two situations were examined. Due to the nature of the equations, there could be infinitely many solutions that differ from the analysis performed here. In order to get a better understanding, further research could investigate other situations, such as each reactor being a different temperature. Additionally, in the analysis the dual root equation used the same equations as the situation when the temperature of the reactor was the steady state temperature. This may not be the best analysis. Further studies could look into different methods of solving such a system.

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