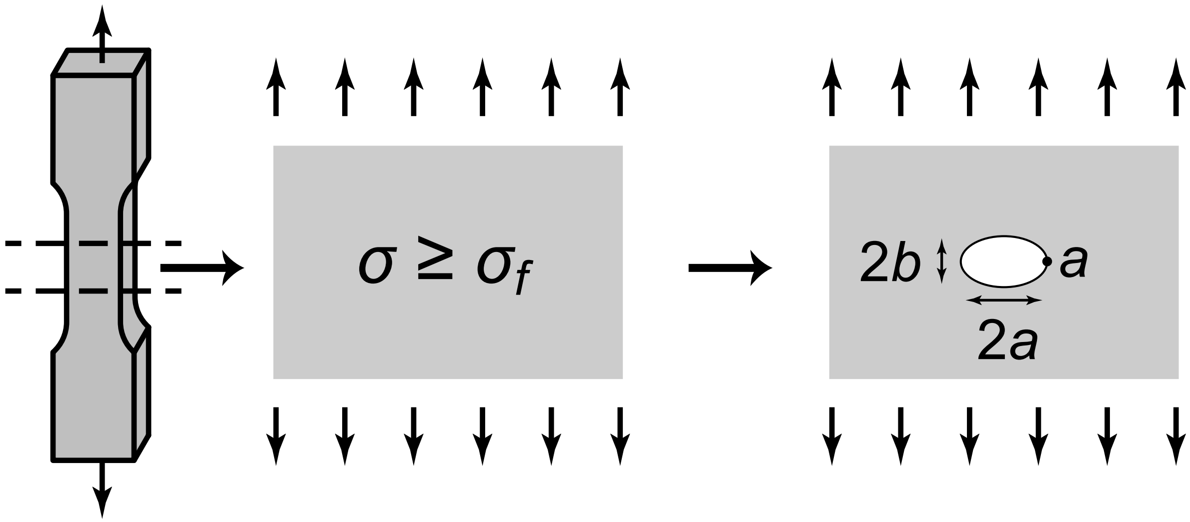
Effects of Pores on Fracture – Stress Concentration –



$\sigma_a = [1+2(a/b)]\sigma$

$\sigma_a = 3\sigma$ (circle)

effect of shape: $k_{conc} = \dfrac{\sigma_{max}}{\sigma}$

effect of size?

Adapted from R.K. Roeder, in: *Characterization of Biomaterials*, Elsevier, pp. 49-104, 2013.

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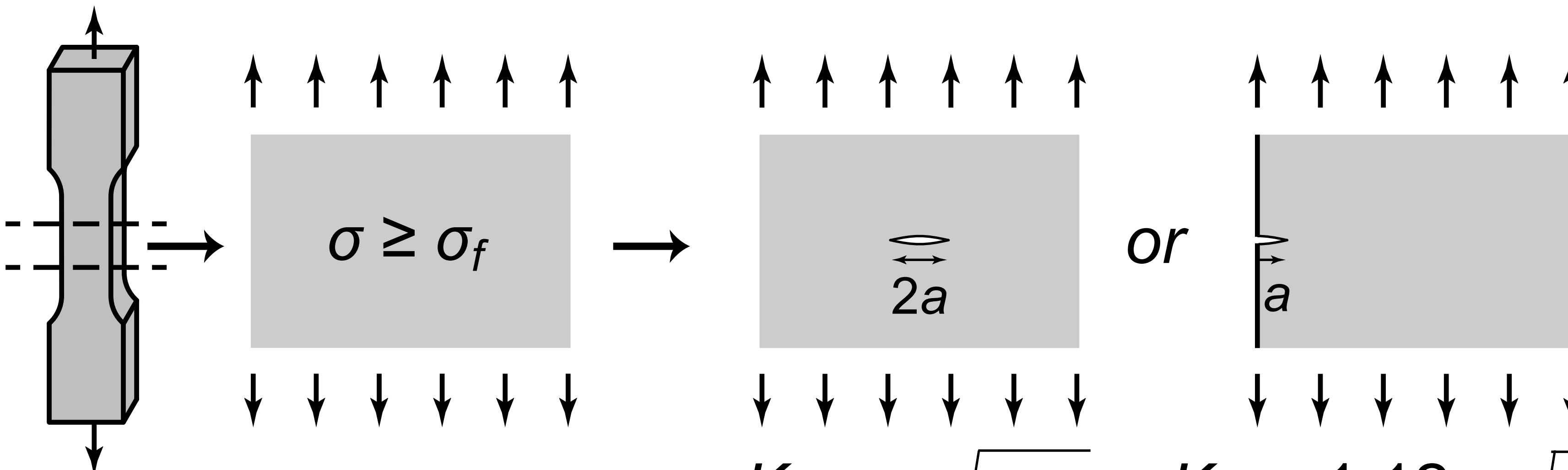
Effects of Pores on Fracture
– Stress Intensity Factor, *K* –

K characterizes stress conditions near crack tip and therefore governs crack initiation.

$K = Y\sigma\sqrt{\pi \cdot a}$ $\quad$ $Y = f$(geometry and loading mode)

K_{Ic} = measure of fracture toughness (material property) in mode I (crack opening) loading.

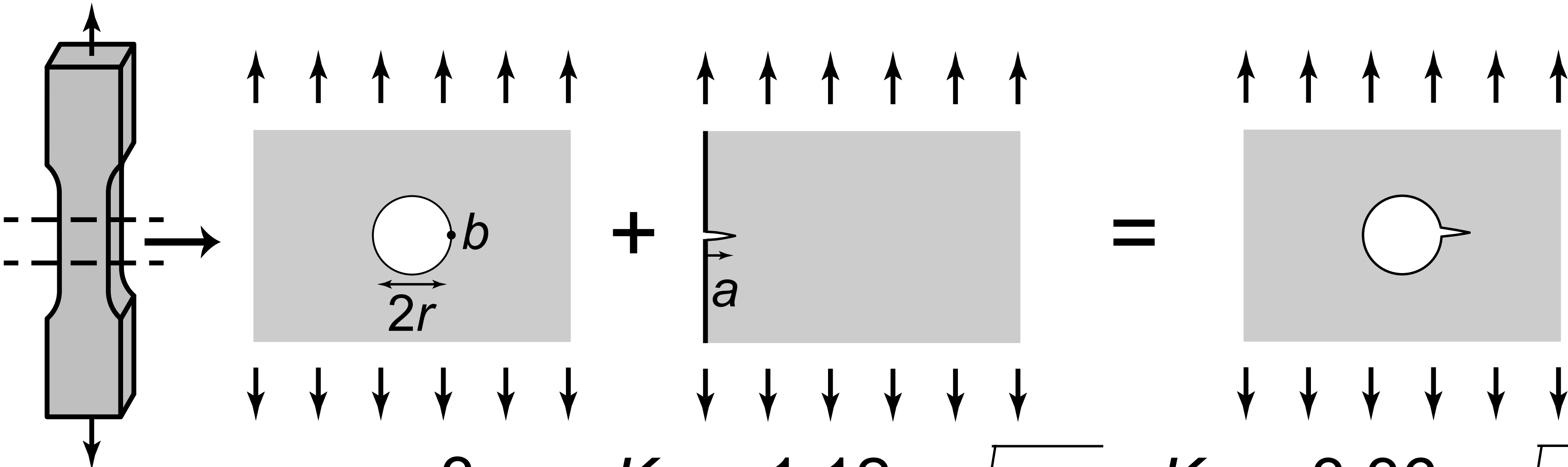


$K_{Ic} = \sigma_f\sqrt{\pi \cdot a}$ $\quad$ $K_{Ic} = 1.12 \cdot \sigma_f\sqrt{\pi \cdot a}$

Adapted from R.K. Roeder, in: *Characterization of Biomaterials*, Elsevier, pp. 49-104, 2013.

Effects of Pores on Fracture – Stress Intensity Factor, K –

principle of superposition!



$\sigma_b = 3\sigma$ $\quad K_{Ic} = 1.12 \cdot \sigma_f \sqrt{\pi \cdot a}$ $\quad K_{Ic} = 3.36 \cdot \sigma_f \sqrt{\pi \cdot a}$

(if $a << r$)

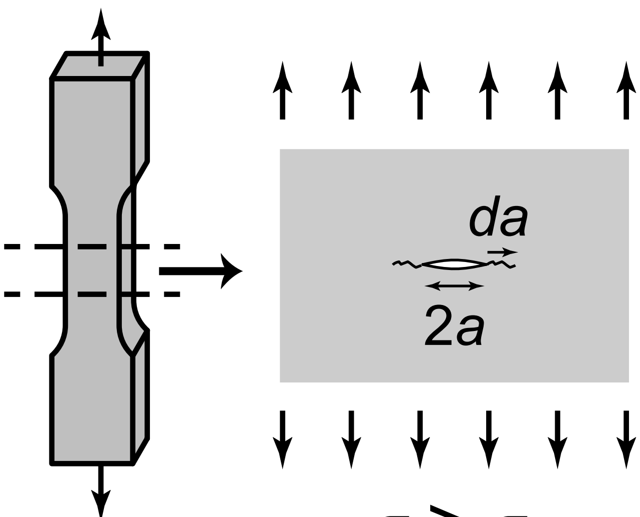
Adapted from R.K. Roeder, in: *Characterization of Biomaterials*, Elsevier, pp. 49-104, 2013.

Effects of Pores on Fracture – Strain Energy Release Rate, $\mathcal{G}$ –

$\mathcal{G}$ characterizes a global energy balance and therefore governs crack propagation.

Griffith's Law (1920)

total energy = strain energy + created surface energy



$\sigma \geq \sigma_f$

$$U(a) = \frac{-\pi \cdot a^2 \cdot b \cdot \sigma^2}{E'} + 4a \cdot b \cdot \gamma$$

$$\frac{dU}{dA} = \frac{-\pi \cdot a \cdot \sigma^2}{E'} + 2 \cdot \gamma = 0$$

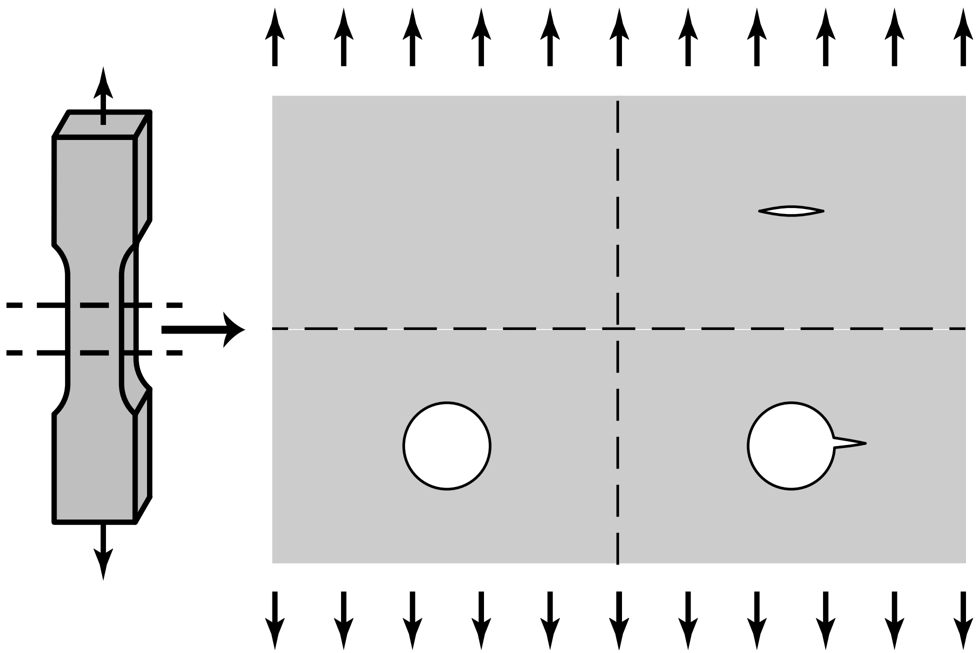
$$\sigma_f = \left(\frac{2E' \cdot \gamma}{\pi \cdot a_c} \right)^{1/2} \quad or \quad E' \cdot 2\gamma = \sigma_f \sqrt{\pi \cdot a}$$

$$\sqrt{E \cdot \mathcal{G}} = K$$

Adapted from R.K. Roeder, in: *Characterization of Biomaterials*, Elsevier, pp. 49-104, 2013.

Effects of Pores on Fracture – Probabilistic Approach –

Larger pores (greater volume) are more likely to intersect other defects (e.g., cracks, inclusions, pores, free surfaces).



$\sigma \geq \sigma_f$

Weibull Statistics

$$P_f = 1 - \exp\left(-\left(\frac{\sigma_f - \sigma_{min}}{\sigma_o}\right)^m\right)$$

$$\frac{\sigma_{f1}}{\sigma_{f2}} = \left(\frac{V_2}{V_1}\right)^{1/m} = \left(\frac{r_2^3}{r_1^3}\right)^{1/m}$$

m = Weibull modulus

$\uparrow m$ = narrower σ_f distr.

Adapted from R.K. Roeder, in: *Characterization of Biomaterials*, Elsevier, pp. 49-104, 2013.