Homework #3
Due: 9/22/16

1. (10) Derive an equation for σ_{11} for an arbitrary transformation of a plane stress state using tensor notation and show that the equation is that same as that provided by Mohr's circle.

2. (50) In a polycrystalline solid of copper (see Table 2.8), suppose that the volume of individual crystals with their <001> crystallographic axes at an angle, χ, from the specimen z-axis are hypothetically found to follow either,

$$f_1(\chi) = 1 \qquad \text{(random distribution), or}$$

$$f_2(\chi) = 3 \cdot \exp(-2.8638 \cdot \chi^2) \text{ (preferred orientation in the specimen } z\text{-axis).}$$

   Note that in the latter case the material is axisymmetric along the z-axis (or a plane of symmetry in the x-y) and the area under each function for χ = 0 to π/2 is equal. Thus, the volume fraction of crystals within a differential element at χ would be,

$$\frac{dV}{V}(\chi) = \frac{f(\chi) \cdot d\chi}{\int_0^{\pi/2} f(\chi) \cdot d\chi}$$

   (a) Find the Voigt, Reuss and Hill average stiffness coefficients for the polycrystal in the specimen z-axis, $\overline{C}_{33}$, for both the random and oriented distribution.
   (b) Plot the effective polycrystal stiffness coefficient, $\overline{C}_{33}$, versus the misorientation angle, χ, from the specimen z-axis for both the random and oriented distribution.
   (c) For your plot in (b), why is the maximum $\overline{C}_{33}$ for the oriented distribution *not* along the specimen z-axis?

3. (20) What effect(s) might you expect grain boundaries to have when averaging single crystal elastic constants to determine the effective polycrystal elastic constants? How could one account for the effect(s) of grain boundaries?

4. (20) A single ply (lamina) of a continuous graphite fiber reinforced epoxy composite is loaded in uniaxial tension (e.g., $\sigma_{xx} \neq 0$, $\sigma_{yy} = \sigma_{zz} = 0$). The composite properties are E_1 = 140 GPa, E_2 = 10 GPa, G_{12} = 10 GPa, and ν_{12} = 0.3. (a) Plot γ_{xy} as a function of the fiber orientation θ. (b) Plot $\eta_{x,xy}$ as a function of the fiber orientation θ.