DEFORMATION SOLUTIONS

Given/Resultant Conditions

- geometry and dimensions, $\varepsilon = \ln\left(\frac{l}{l_o}\right) = \ln\left(\frac{A_o}{A}\right) =$ etc.
- applied/resultant forces, $F = \sigma \cdot A$, pressures, torques, work, $u = volume \cdot \int_0^{\varepsilon} \sigma \cdot d\varepsilon$, etc.

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Applied Stress State

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

↓↑

Stress Transformation (Rotation of Axes)

- Mohr's Circle
- Tensor Rotation

↓↑

Principal Stresses and Strains

$$\begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} \text{ and } \begin{bmatrix} \varepsilon_1 & & \\ & \varepsilon_2 & \\ & & \varepsilon_3 \end{bmatrix}$$

↓↑

Effective Stress and Strain

Tresca (maximum shear stress):

$\bar{\sigma} = \sigma_3 - \sigma_1$ (if $\sigma_3 > \sigma_2 > \sigma_1$)

$\bar{\varepsilon} = 2/3 \cdot (\varepsilon_1 - \varepsilon_3)$

von Mises (distortion energy):

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

$$\bar{\varepsilon} = \frac{\sqrt{2}}{3}\left[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2\right]^{1/2}$$

Friction (Slab Method)

Forging: $F = p_{avg} \cdot 2aw$

$p_{avg} = Y' \cdot (1 + \mu a/h)$, $p = Y' \cdot \exp[2\mu(a - x)/h]$ (sliding)

$p_{avg} = Y' \cdot (1 + a/2h)$, $p = Y' \cdot [1 + (a - x)/h]$ (sticking)

Rolling: $F \cong \overline{Y'} \cdot w\sqrt{R \cdot \Delta h}$ (low friction)

$F \cong \overline{Y'} \cdot w\sqrt{R \cdot \Delta h} \cdot \left(1 + \frac{\mu\sqrt{R \cdot \Delta h}}{2 \cdot h_{avg}}\right)$ (high fiction)

$p = Y_f' \cdot \frac{h}{h_o} \cdot \exp[\mu(H_o - H)]$ (entry zone)

$p = Y_f' \cdot \frac{h}{h_o} \cdot \exp[\mu H]$ (exit zone)

$$H = 2\sqrt{\frac{R}{h_f}} \cdot \tan^{-1}\left(\sqrt{\frac{R}{h_f}} \cdot \phi\right)$$

Extrusion: $p = Y \cdot [1 + \tan\alpha/\mu] \cdot [R^{\mu \cdot \cot\alpha} - 1]$ (die only)

$p \cong 1.7 \cdot Y \cdot \ln R$ (die only)

$p \cong Y \cdot (1.7 \cdot \ln R + 2L/D_o)$ (die & container)

Drawing: $\sigma_d = Y \cdot [1 + \tan\alpha/\mu] \cdot [1 - (A_f/A_o)^{\mu \cdot \cot\alpha}]$

$\sigma_d = \bar{Y} \cdot [1 + \alpha/\mu] \cdot \ln(A_f/A_o) \cdot \Phi$ (inhomog.)

↓↑

Material Properties (Plastic)

$\sigma = Y$ perfectly plastic, "ideal"

$\sigma = K \cdot \varepsilon^n$ strain hardening (power law) $\bar{\sigma} = K \cdot \bar{\varepsilon}^n$

note: strain rate effects could also be included here

↓↑

Yield Criteria

yielding occurs when… $\bar{\sigma} = Y = 2 \cdot k$ (Tresca)

$\bar{\sigma} = Y = \sqrt{3} \cdot k$ (von Mises)

Y = yield strength, k = shear yield strength, $Y' = 2Y/\sqrt{3}$ = plane 🎁 Y

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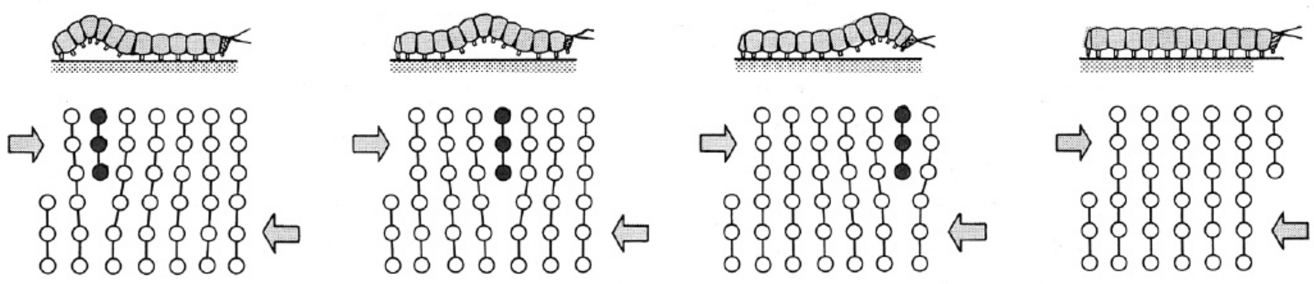
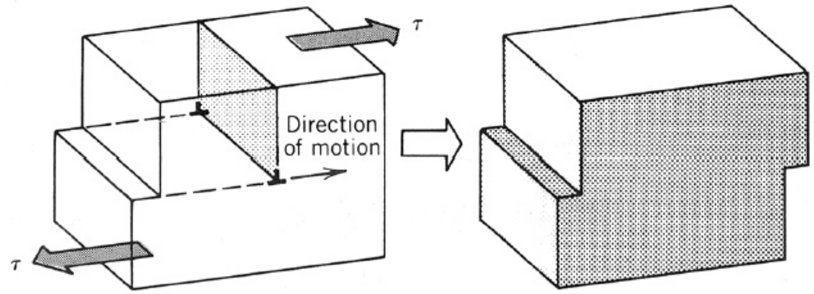
Dislocations

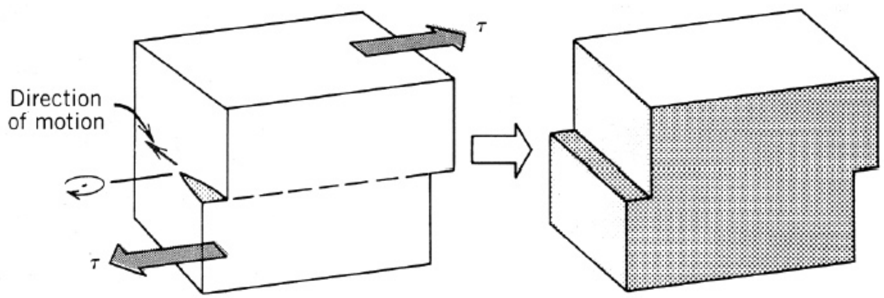
motion analogy:



edge:



screw:



W.D. Callister, Jr., *Materials Science and Engineering*, 2nd Ed., John Wiley and Sons, New York, 1991.

Plastic Anisotropy: Deep Drawing

Figure 14-2 Drawing failures by necking at bottom of cup wall. With very low friction, the failure site tends to move onto the punch radius as shown at the right. From D. J. Meuleman, Ph.D. thesis, Univ. of Michigan (1980).

Figure 14-3 Wrinkling in a partially drawn cup due to insufficient hold-down force. From D. J. Meuleman, *ibid.*

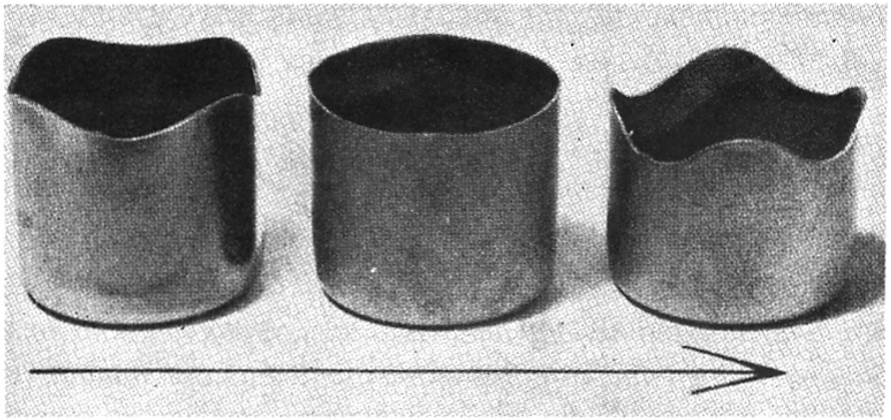


Figure 14-12 Earing behavior of cups made from three different copper sheets. Arrow indicates rolling direction of the sheets. From D. V. Wilson and R. D. Butler, *J. Inst. Met.*, 90 (1961–2), pp. 473–83.

W.F. Hosford and R.M. Caddell, *Metal Forming*, 2nd Ed., Prentice-Hall, Inc., Edgewood Cliffs, NJ, 1993.

Plastic Anisotropy: Deep Drawing

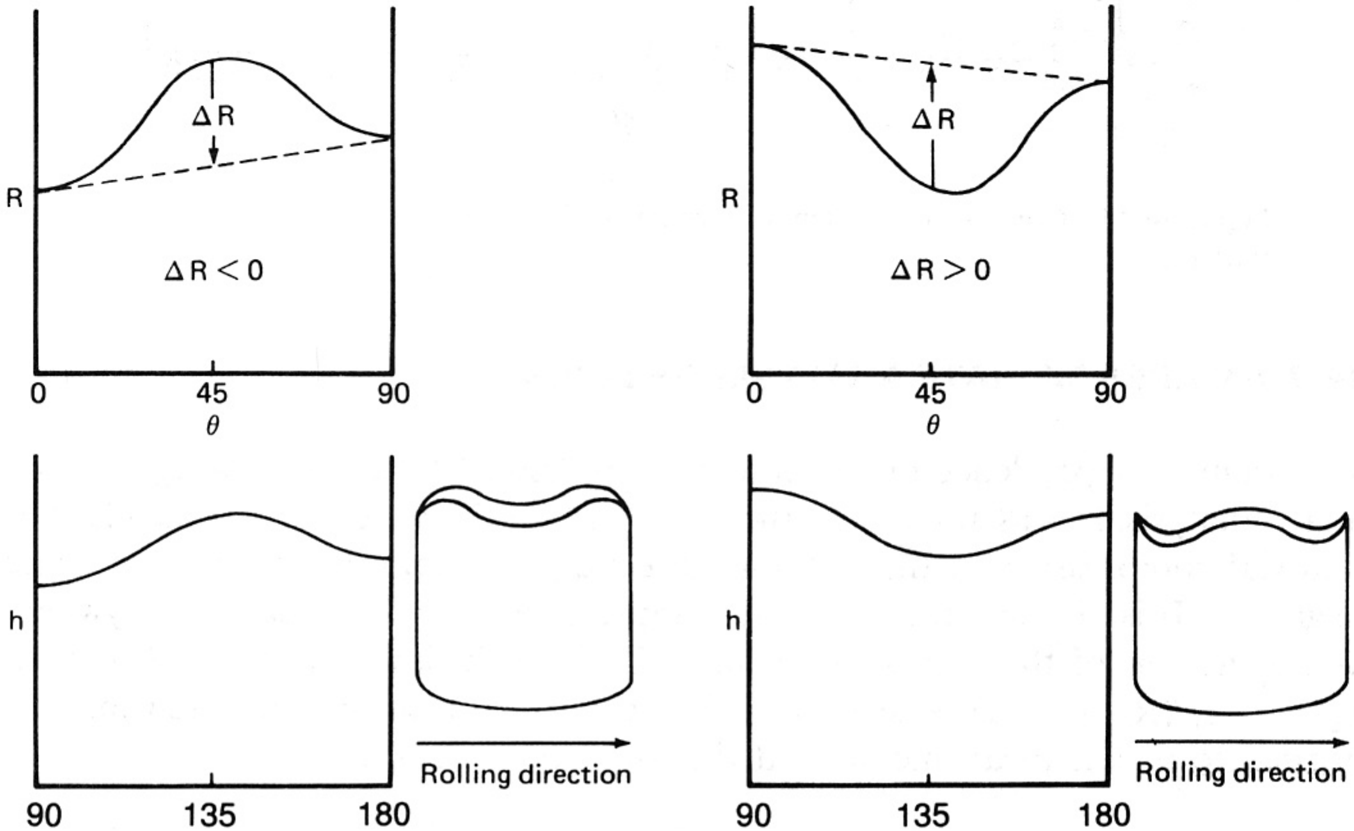
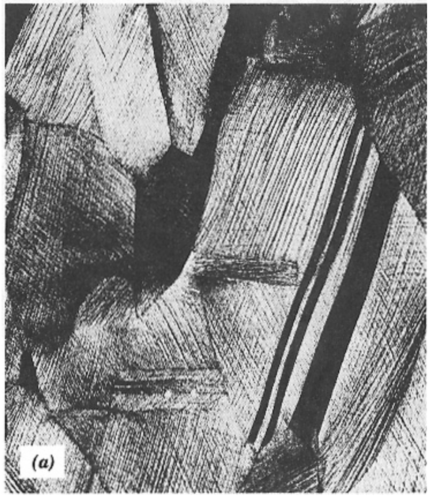


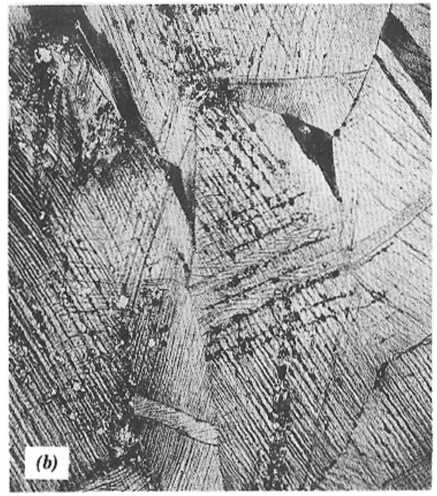
Figure 14-13 Relation of earing to angular variations of *R*. Here, *h* is the wall height.

W.F. Hosford and R.M. Caddell, *Metal Forming*, 2nd Ed., Prentice-Hall, Inc., Edgewood Cliffs, NJ, 1993.

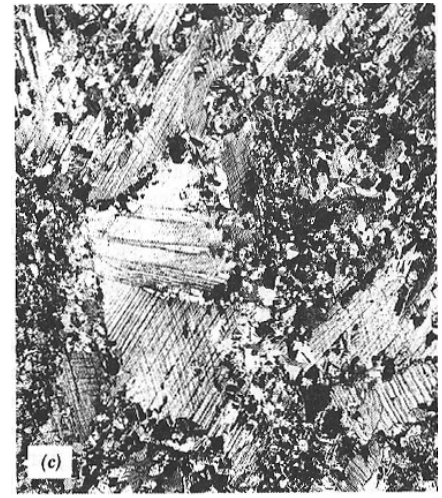
Recovery, Recrystallization and Grain Growth



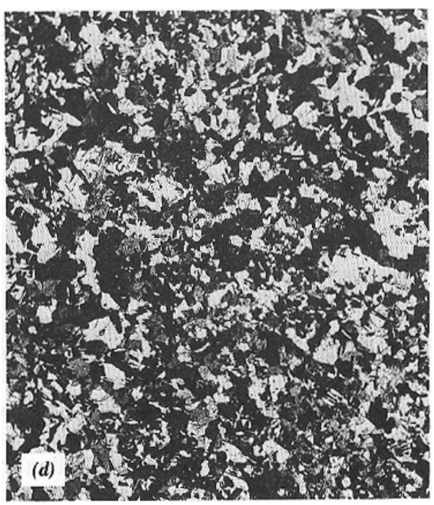
cold worked brass



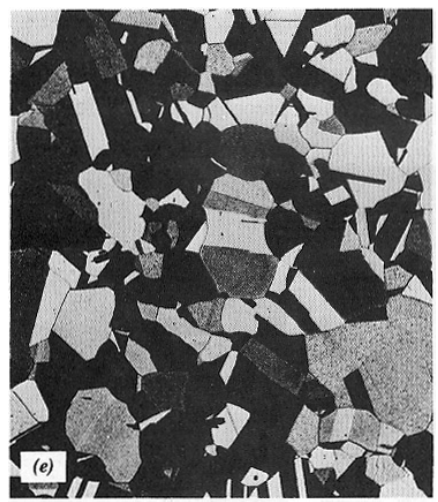
early recrystallization
(3 s at 580°C)



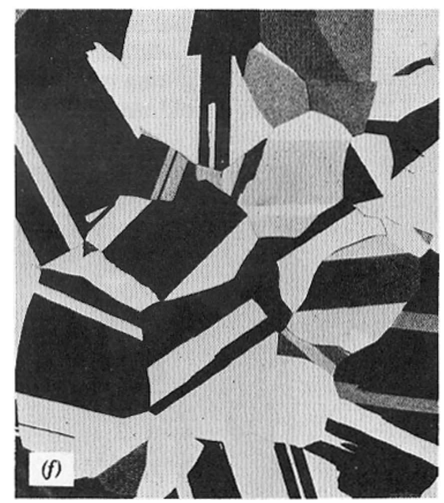
partial recrystallization
(4 s at 580°C)



full recrystallization
(8 s at 580°C)

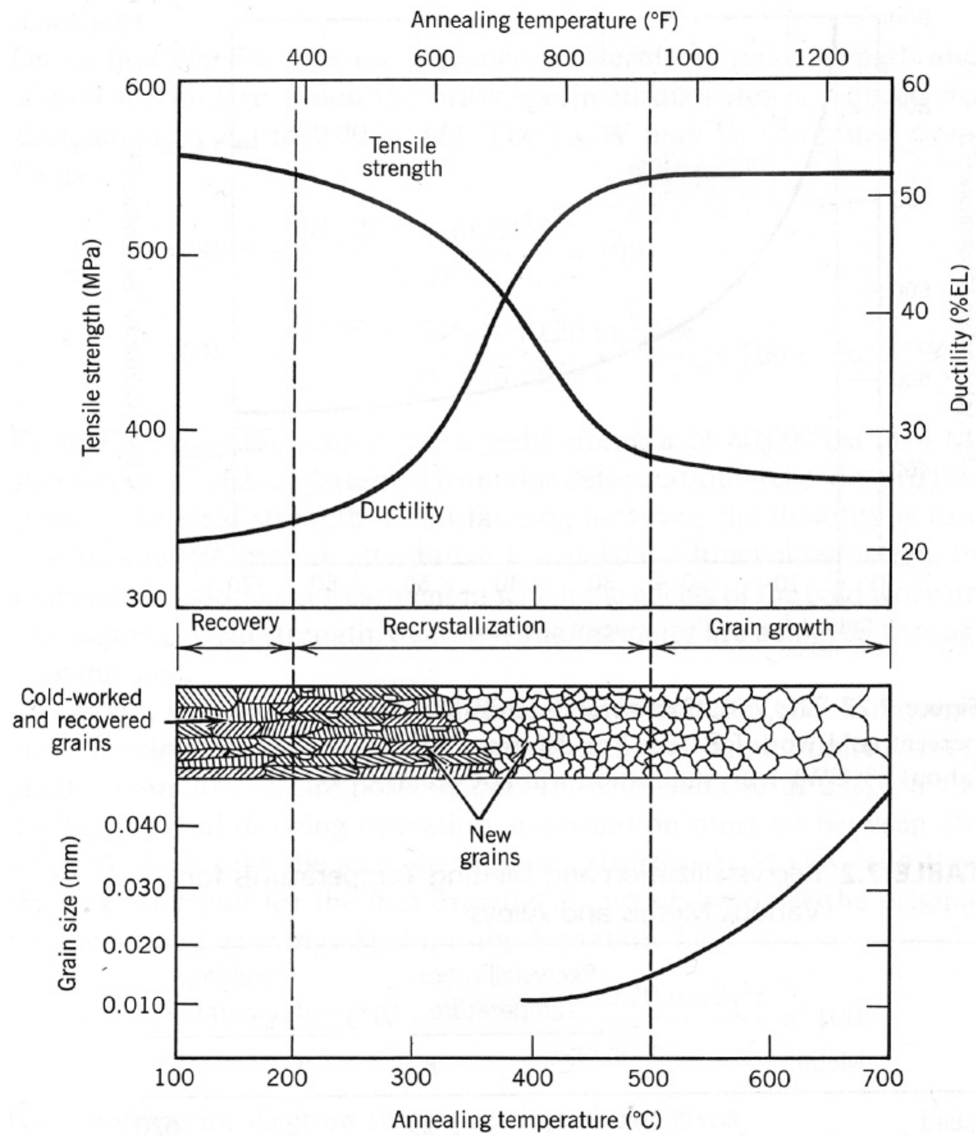


grain growth
(15 min at 580°C)



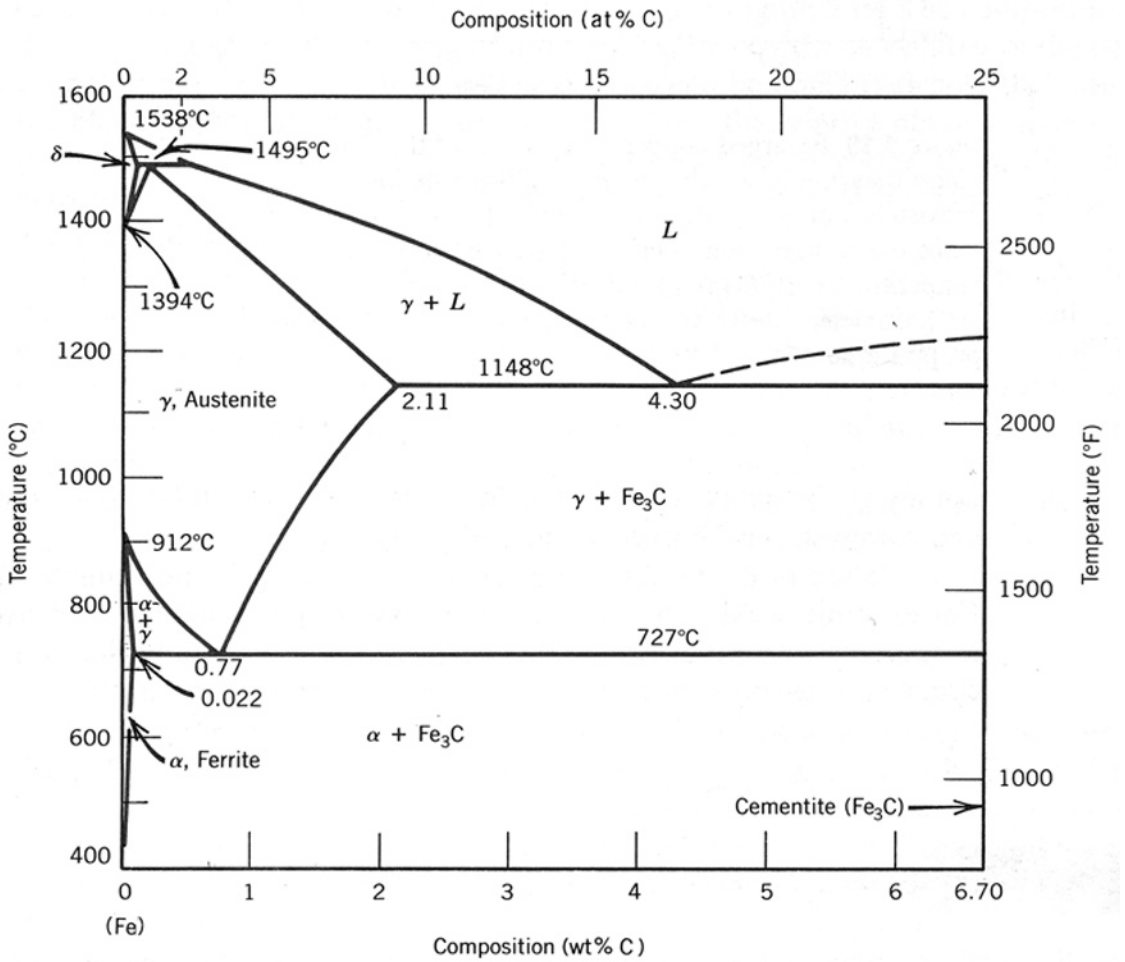
grain growth
(10 min at 700°C)

Recovery, Recrystallization and Grain Growth



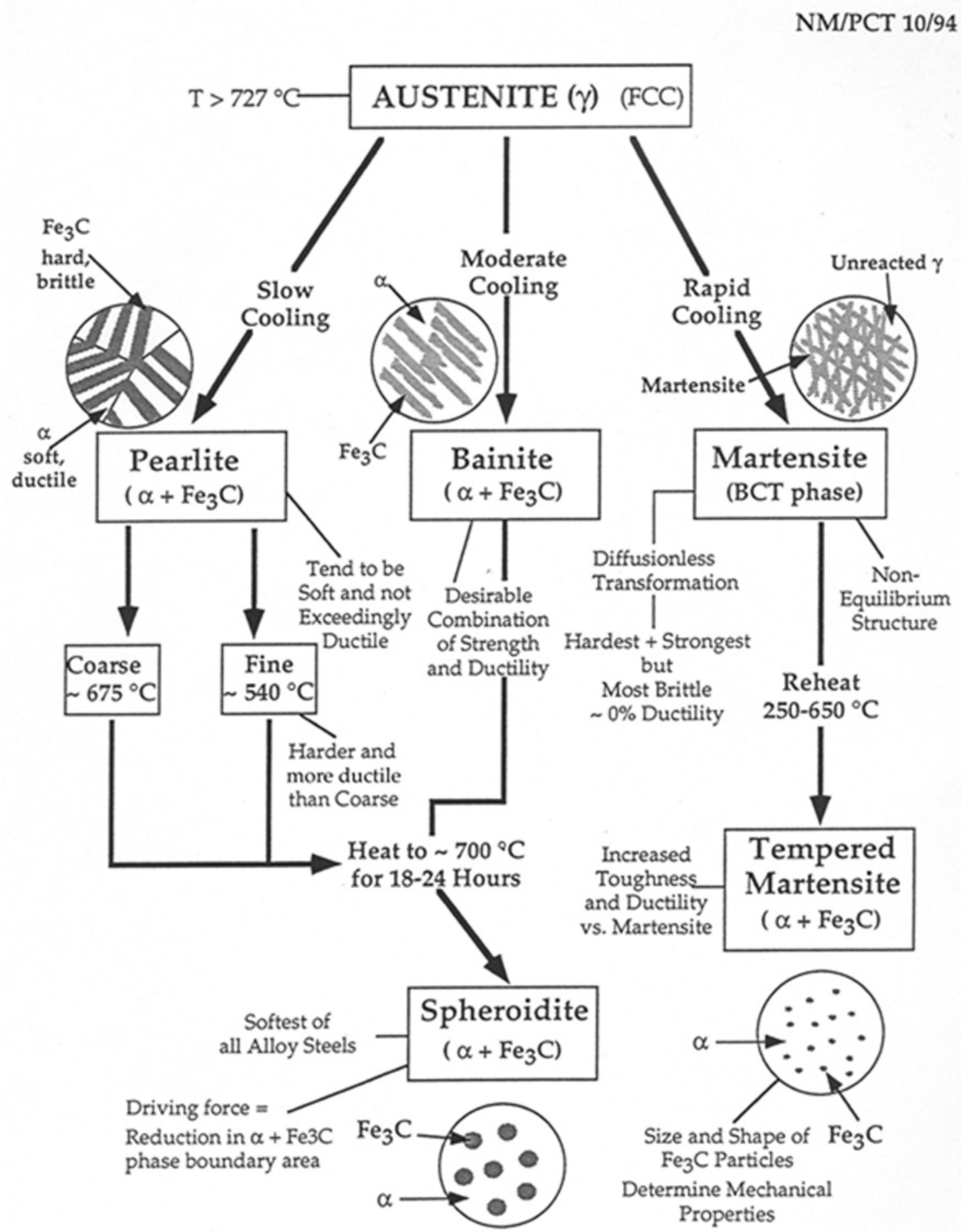
W.D. Callister, Jr., *Materials Science and Engineering*, 2nd Ed., John Wiley and Sons, New York, 1991.

Strengthening Mechanisms: Steel Structures

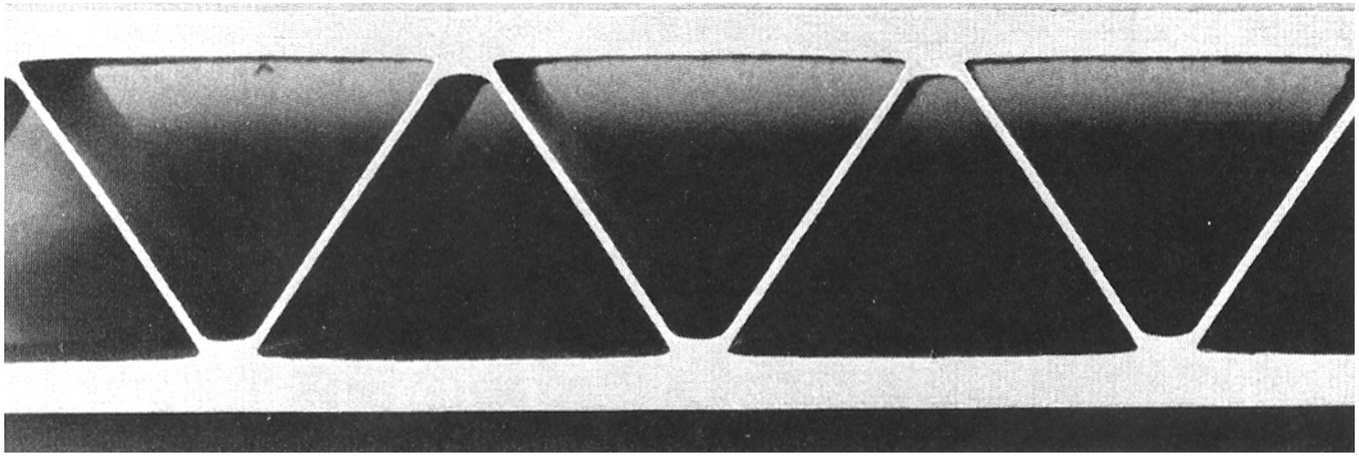


Adapted from: *Metals Handbook: Metallography, Structures and Phase Diagrams*, Vol. 8, 8th edition, T. Lyman Ed., American Society for Metals, Materials Park, OH, 1973.

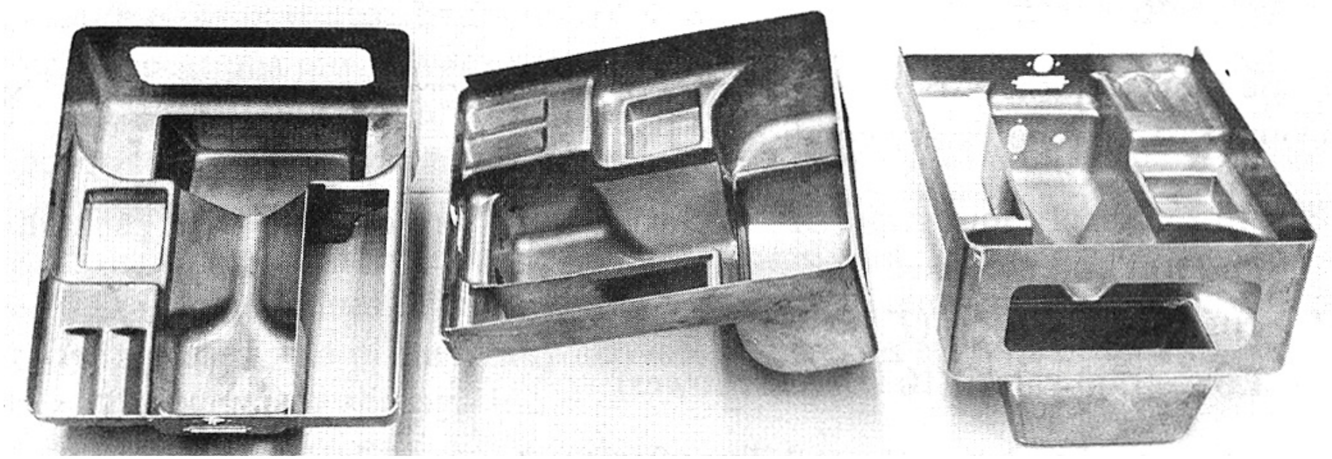
Strengthening Mechanisms: Steel Structures



Superplastic Forming



Titanium alloy aircraft panel produced by diffusion bonding followed by superplastic expansion using internal pressure. Courtesy of Rockwell International Corp.



Complex shapes formed from Zn-22%Al sheet metal using superplastic forming. Courtesy of D.S. Fields, IBM Corp.

W.F. Hosford and R.M. Caddell, *Metal Forming*, 2nd Ed., Prentice-Hall, Inc., Edgewood Cliffs, NJ, 1993.