Weibull Statistics Examples

All examples pasted from a Mathcad™ worksheet. All stresses in MPa.

1) Weibull distribution of fracture stresses measured for 30 specimens in uniaxial tension

$i := 1..30$ $j_i := i$

$\sigma_i :=$

187
228
243
261
289
298
311
321
343
359
366
371
373
381
390
395
404
406
410
419
422
435
442
449
458
466
482
500
508
529

m estimated from a normal distribution:

$mean(\sigma) = 381.5$ $stdev(\sigma) = 84.2$

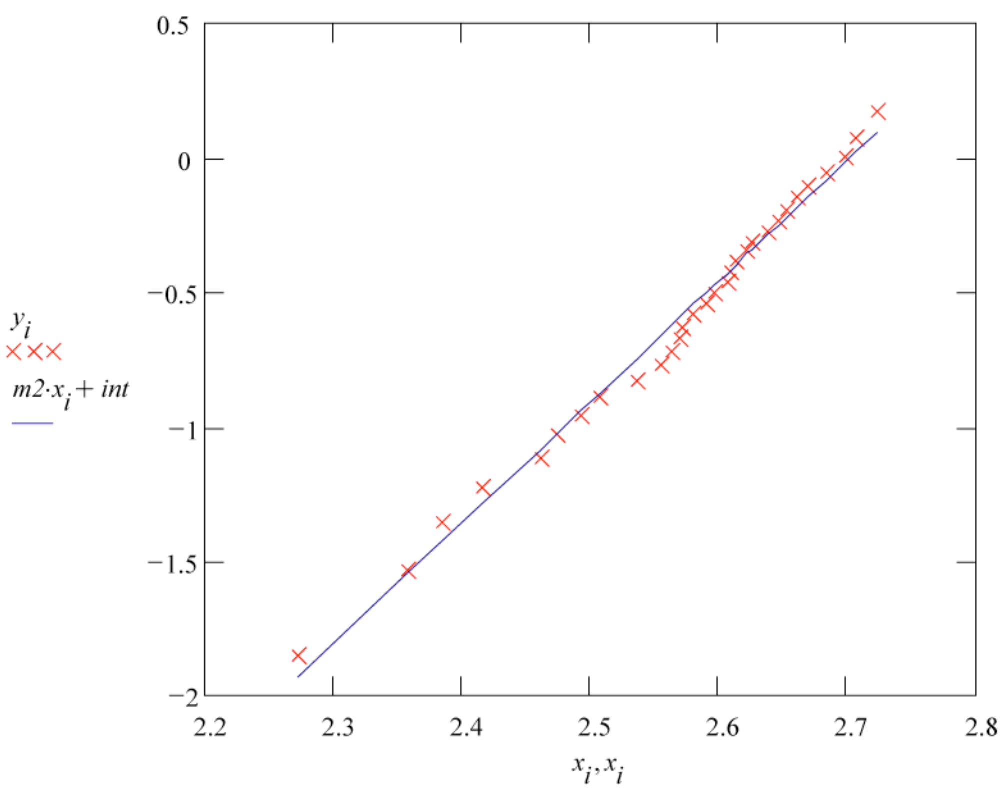
$m1 := \frac{1.2 \cdot mean(\sigma)}{stdev(\sigma)}$ $m1 = 5.4$

m determined from Weibull plot: $x_i := log(\sigma_i)$

Weibull Parameter = $y_i := log\left[\left[log\left[\frac{max(j)+1}{max(j)-j_i+1}\right]\right]\right]$

$m2 := slope(x,y)$ $m2 = 4.5$

$int := intercept(x,y)$ $int = -12.2$



2) Three-point bending vs. uniaxial tension (above) using the stress volume integral:

$$P_f(m, \sigma, \sigma_{mean}, V, V_{ref}, SVI) := 1 - exp\left[-\left[\Gamma\left[\frac{1}{m} + 1\right]\right]^m \cdot \left[\frac{\sigma}{\sigma_{mean}}\right]^m \cdot \frac{V}{V_{ref}} \cdot SVI\right]$$

stress volume integral (3pt bending): $SVI(m) := \frac{1}{2 \cdot (m+1)^2}$

$V_{3ptbend} := 4 \cdot 2 \cdot 0.25$ $V_{3ptbend} = 2$ mm³ $V_{tensile} := 4 \cdot \pi \cdot 0.5^2$ $V_{tensile} = 3.1$ mm³

For the same maximum stress in the bend bar as the average of the tensile data:

$$P_f(m2, mean(\sigma), mean(\sigma), V_{3ptbend}, V_{tensile}, SVI(m2)) = 6.95 \cdot 10^{-3}$$

In other words, for these conditions, there is a less than 1% chance of failure in bending even though half of the tensile specimens would fail at this level. Furthermore, for the same conditions, but a much higher Weibull Modulus, the probability is even less for this stress.

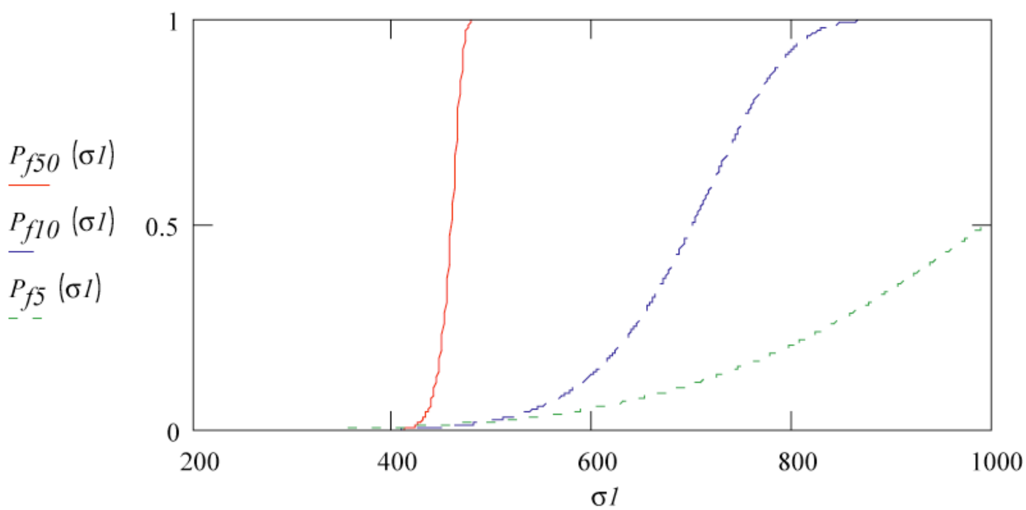
$$P_f(10, mean(\sigma), mean(\sigma), V_{3ptbend}, V_{tensile}, SVI(10)) = 1.596 \cdot 10^{-3}$$

3) The effects of changing *m* on the distribution of failure stresses in bending:

$\sigma 1 := 1, 2 .. 1000$ $P_{f5}(\sigma 1) := P_f(5, \sigma 1, mean(\sigma), V_{3ptbend}, V_{tensile}, SVI(5))$

$P_{f10}(\sigma 1) := P_f(10, \sigma 1, mean(\sigma), V_{3ptbend}, V_{tensile}, SVI(10))$

$P_{f50}(\sigma 1) := P_f(50, \sigma 1, mean(\sigma), V_{3ptbend}, V_{tensile}, SVI(50))$

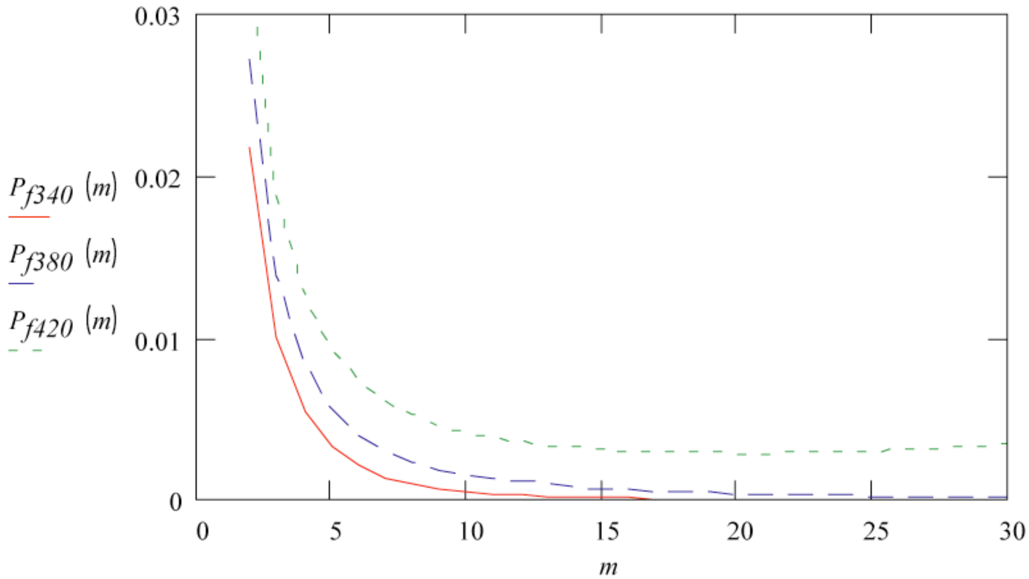


4) The effects of changing *m* on the stresses and the gamma function on the failure probability:

$$m := 2..30 \qquad P_{f340}(m) := 1 - exp\left[-\left[\Gamma\left[\frac{1}{m}+1\right]\right]^m \cdot \left[\frac{340}{mean(\sigma)}\right]^m \cdot \frac{V_{3ptbend}}{V_{tensile}} \cdot SVI(m)\right]$$

$$P_{f380}(m) := 1 - exp\left[-\left[\Gamma\left[\frac{1}{m}+1\right]\right]^m \cdot \left[\frac{380}{mean(\sigma)}\right]^m \cdot \frac{V_{3ptbend}}{V_{tensile}} \cdot SVI(m)\right]$$

$$P_{f420}(m) := 1 - exp\left[-\left[\Gamma\left[\frac{1}{m}+1\right]\right]^m \cdot \left[\frac{420}{mean(\sigma)}\right]^m \cdot \frac{V_{3ptbend}}{V_{tensile}} \cdot SVI(m)\right]$$



5) The effects of the volume and stress volume integral ratio on the average failure stress:

$$stressratio(V1, V2, SVI1, SVI2, m) := \left[\frac{V1 \cdot SVI1}{V2 \cdot SVI2}\right]^{\frac{1}{m}}$$