

Fracture resistance of paper

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An attempt has been made to apply the concepts of fracture mechanics to describe the behaviour of a paper sheet with a crack. Considering paper as an orthotropic homogeneous continuum, the critical strain energy release rate, G_c , for eight different papers has been measured using linear elastic fracture mechanics. Also, a direct measurement of work of fracture, R , has been made for these samples by using the quasi-static crack propagation technique. For both techniques, results independent of specimen dimensions were obtained. G_c and R were found to be experimentally equivalent. The fracture toughness of paper has been compared with that of other engineering materials.

Nomenclature

- a Initial crack length (cm)
 a_{ij} Elements of compliance matrix \bar{a} ($\text{cm}^2 \text{dyn}^{-1}$)
 A Area of fractured surface (cm^2)
 b Specimen width (cm)
 E Young's modulus (dyn cm^{-2})
 E_1 Young's modulus in the machine direction (dyn cm^{-2})
 E_2 Young's modulus in the cross direction (dyn cm^{-2})
 E_θ Young's modulus at angle θ to the machine direction (dyn cm^{-2})
 F Finite-width correction factor
 G Strain energy release rate (erg cm^{-2})
 G_c Critical strain energy release rate (erg cm^{-2})
 K Stress intensity factor ($\text{dyn cm}^{-3/2}$)
 K_c Critical stress intensity factor ($\text{dyn cm}^{-3/2}$)
 l Specimen length (cm)
 r_y Size of plastic zone (cm)
 R Work of fracture (erg cm^{-2})
 t Specimen thickness (cm)
 U Strain energy (erg)
 θ Angle in the plane of the sheet measured from the machine direction
 ρ Specimen density (g cm^{-3})
 σ_c Gross tensile stress at fracture (dyn cm^{-2})
 σ_N Net tensile stress at fracture (dyn cm^{-2})
 σ_{ys} Tensile yield stress (dyn cm^{-2})

1. Introduction

Paper is generally made from wood, which

consists of a parallel array of cellulosic fibres bound together by the polymer, lignin. The fibres are hollow tubes typically 1 to 3 mm long, 25 to 35 μm wide and of wall thickness 1 to 4 μm . During pulping the fibres are separated either by mechanical or chemical means and dispersed in water. The paper sheet is formed by filtering this pulp suspension followed by pressing and drying. During drying bonds form at fibre junctions between the hydrophilic surfaces. A paper sheet thus consists of a dense two-dimensional array of fibres, bound together at regions where they cross. The strength of a paper sheet arises from the strength of fibres and the number and strength of the fibre-fibre bonds.

The properties of commercial papers are generally anisotropic, because of a preferred orientation of fibres in the direction of manufacture*, and the tension exerted on the continuous web in this direction during drying. For stresses parallel to the plane of the sheet, it has been shown [1] that paper behaves elastically as an orthotropic material.

When a small specimen of paper is loaded under tension, it exhibits elastic behaviour followed by inelastic deformation before failure. A typical load-elongation curve for paper is shown in Fig. 1. The area under this curve is the work done to break the specimen. A part of this work is consumed by the specimen in producing permanent deformation in the form of broken interfibre bonds [2] and removal of microcrimps

*This direction is referred to as the machine direction (M.D.) and the direction at right angles as the cross direction (C.D.)

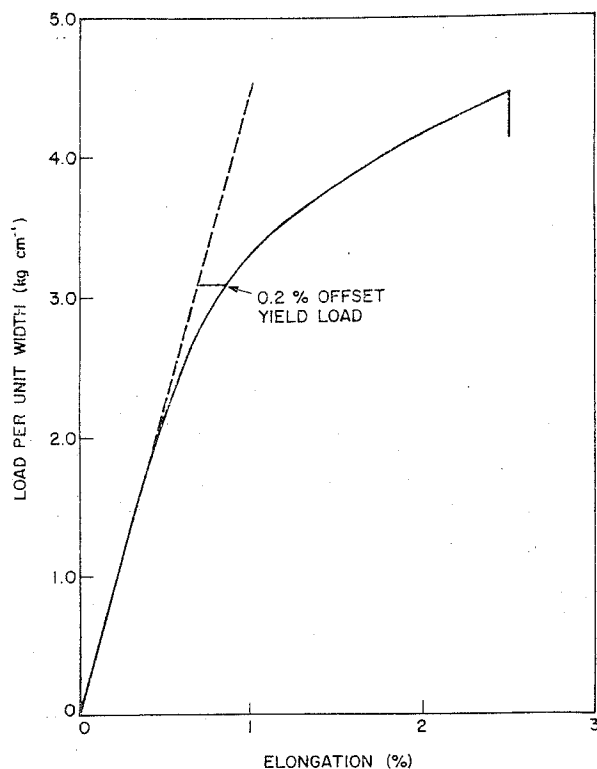


Figure 1 A typical load-elongation curve for an unflawed paper sheet.

[3] in the fibres. The other part provides the recoverable strain energy immediately prior to rupture, and this is dissipated in fracture of the specimen and recoil of the broken ends.

During manufacture or in subsequent converting operations, when a sheet is subjected to tensile stress, it sometimes fails. It has been shown [4, 5] that these failures often occur by rapid propagation of a pre-existing flaw such as an unpulped particle of wood, or a region damaged at some stage in the process. Because of stress concentration around the flaw, failure occurs at a stress well below the strength of an unflawed sheet.

The techniques of fracture mechanics are aimed at this problem – the definition and measurement of a material property, fracture resistance, which is the ability of a material to resist crack propagation from a pre-existing flaw. These techniques have been successfully applied to metals, polymers, and composites. Some preliminary work on the application of fracture mechanics to paper has been carried out by Balodis [6] and Andersson and Falk [7], and this will be discussed later. The purpose of this study is to further these investigations in the light of recent developments in fracture mechanics.

The fracture resistance of paper has been measured following two different approaches. In the first, linear elastic fracture mechanics (LEFM) techniques have been used, considering paper as an orthotropic homogeneous continuum. In the second, the work of fracture has been measured directly by quasi-static [8] propagation of a crack in a paper sheet. The results obtained from the two techniques should be in agreement as predicted theoretically by Gurney and Hunt [8].

2. Theoretical considerations

For a body containing a crack, the Griffith energy balance criterion for crack growth is

$$G = - \left[\frac{\partial U}{\partial A} \right]_l, \quad (1)$$

where G is the strain energy release rate for a fixed length l of the specimen, U is the elastic energy stored in the body, and A is the area of the fractured surface. A crack will propagate if the energy that is available equals or exceeds the characteristic energy required to create the crack, i.e.

$$G \geq G_c. \quad (2)$$

The two quantities G and G_c are quite distinct. The former is a function of the geometry of the body, its elastic properties and loading conditions, while the latter is the energy absorbed by the material in the process of crack extension and is a material property. G_c can be used to characterize the crack resistance of the material. It is common practice to measure G_c which is the critical value of G at which crack extension is observed experimentally.

2.1. Fracture mechanics approach

Fracture mechanics offers a method for evaluating the critical strain energy release rate G_c . It consists of characterization of the crack-tip stress field in terms of a factor K , the stress intensity factor. K describes the state of stress in the vicinity of the crack tip as a function of the specimen geometry, the crack geometry and the applied load.

For an elastic specimen of width $2b$ containing a crack and under a tensile stress σ_c at which instability occurs, the critical stress intensity factor K_c in the plane stress opening mode is given by [9]

$$K_c = \sigma_c \sqrt{(\pi a) F(a/b)} \quad (3)$$

where $2a$ is the crack length at instability. $F(a/b)$ is a finite-width correction factor and is unity for an infinite plate.

The elastic stress analysis is valid for brittle materials and K_c is found to be independent of crack length. However, most materials of interest exhibit plastic deformation at the crack tip prior to fracture. For small scale yielding, the elastic stress intensity approach can still be applied provided a correction is made for the region around the crack tip where the stress exceeds the yield stress σ_{ys} of the material. This correction, due to Irwin [10], is made by adding to the initial crack length a , a plastic zone of size r_y given, for the plane stress condition, by

$$r_y = \frac{1}{2\pi} \left[\frac{K_c}{\sigma_{ys}} \right]^2 \quad (4)$$

The effect of the correction is to create a stress field identical to the elastic field, but shifted ahead by r_y as if the crack tip were at the centre of the plastic zone of diameter $2r_y$.

For an isotropic material in plane stress, K_c and G_c are related by [11]

$$G_c = K_c^2/E, \quad (5)$$

where E is the elastic modulus. Sih *et al.* [12] have shown that for a homogeneous orthotropic material, the expression for the critical stress intensity factor K_c in the plane stress opening mode remains the same, but the expression for G_c for a crack propagating parallel to a plane of elastic symmetry becomes

$$G_c = K_c^2 \left[\frac{a_{11}a_{22}}{2} \right]^{\frac{1}{2}} \left[\left(\frac{a_{11}}{a_{22}} \right)^{\frac{1}{2}} + \frac{2a_{12} + a_{66}}{2a_{22}} \right]^{\frac{1}{2}} \quad (6)$$

The a_{ij} are elements of the compliance matrix \bar{a} .

The determination of G_c using the linear elastic fracture mechanics approach involves, therefore, measurement of the critical failure stress of a specimen containing a crack of known size, together with measurement of yield stress and the elastic constants.

2.2. Quasi-static crack propagation approach

When the fracture criterion (1) is fulfilled it is energetically possible for the crack to grow [13]. If it should do so, its subsequent growth can be unstable or stable depending on the rate of

change of the energy supply $[-(\partial U/\partial A)]$ as the crack increases in size, i.e. on the second derivative $[-(\partial^2 U/\partial A^2)]$. If $[-(\partial^2 U/\partial A^2)]$ is positive, the energy being released is more than the energy-demand to create new area and the crack will be unstable. For this reason G_c cannot be measured in this case from the energy content of the specimen, but must be determined from a consideration of the stresses at failure as in the approach given above. If, however, $[-(\partial^2 U/\partial A^2)]$ is negative, the energy release becomes less than G_c , the crack is stable and external work is necessary to cause further crack growth. This permits a direct experimental determination of G_c . It is only necessary to use a sample size and crack geometry such that crack growth proceeds slowly as the sample is strained, and to measure the energy transmitted to the sample that produces a unit increase in crack area. Gurney and Hunt [8] have shown that in this so-called "quasi-static" process the energy absorbed, R , is theoretically equal to G_c for a linearly elastic system.

3. Experimental

3.1. Materials and testing conditions

The samples selected for the experimental programme were machine-made papers of different strength and other physical properties. The data on the mechanical properties of the test samples are presented in Table I. All tests were carried out at 73°F (23°C) and 50% relative humidity. Measurement of G_c and R were made for cracks propagating in the cross direction, as this is the usual failure direction during manufacturing and converting.

3.2. Units

Since paper is a porous material with a rough surface, it is not possible to define or measure its thickness in any other than an arbitrary way. The tensile properties are not, therefore, expressed in units of stress but in units of stress per unit sheet density. This requires a measurement only of the breaking load at a known sample width, and the mass of the specimen per unit area. Similarly, the fracture energy is not expressed in the usual units but either in erg cm^{-1} (equivalent to $G_c t$, where t is the real thickness) or erg-cm g^{-1} (equivalent to G_c/ρ , where ρ is the sheet density).

3.3. Measurement of K_c and G_c

A considerable effort was devoted to develop

TABLE I Mechanical properties of test materials

Specimen	Substance (10^{-4} g cm^{-2})	Apparent thickness (mm)	Tensile strength (10^8 dyn cm^{-1})	Elongation-to-break (%)		Elastic modulus (10^{10} dyn cm^{-1})		0.2% offset yield strength (10^8 dyn cm^{-1})	
				M.D.	C.D.	M.D.	C.D.		M.D.
Bond paper 1	73.3	0.101	5.88	4.10	1.15	3.08	8.04	4.94	5.03
Bond paper 2	59.5	0.095	9.07	3.92	1.90	3.52	9.91	4.19	5.96
Tracing paper	56.2	0.070	8.81	5.00	2.66	6.18	9.56	4.69	5.91
Writing paper	57.2	0.085	7.27	3.52	1.75	3.70	8.68	4.23	4.85
Newsprint 1	49.9	0.080	4.41	1.74	0.94	2.39	6.54	1.99	4.21
Newsprint 2	47.5	0.092	5.34	2.28	0.91	2.08	7.38	2.39	5.26
Semibleached kraft paper	49.0	0.060	12.50	4.80	1.21	4.57	14.92	4.45	11.41
Unbleached kraft paper	44.4	0.079	8.30	4.14	1.43	4.45	9.55	3.91	6.64

All results are averages of 10 tests

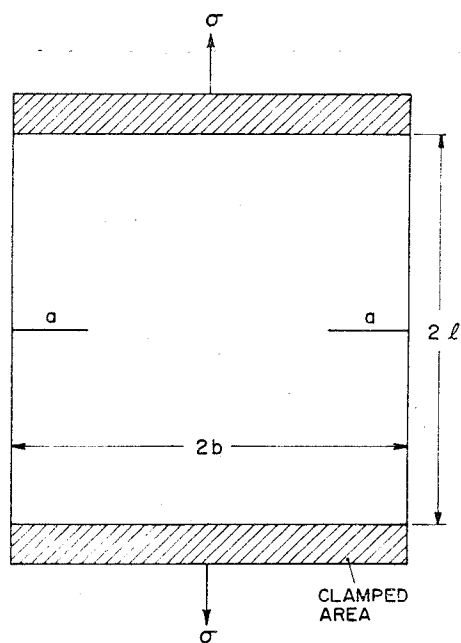


Figure 2 Geometry of specimen used for determination of K_c .

suitable techniques for the application of linear elastic fracture mechanics to paper and to determine the effect of material and testing variables. There is no standard procedure for obtaining K_c under plane stress but two conditions are necessary. Firstly, the net stress over the uncracked region, σ_N , applied to the specimen at failure, should be less than the yield stress [14]. Secondly, the specimen dimensions should be large enough that the boundaries do not interfere with the crack tip stress distribution [15]. These conditions can be met by using a large enough specimen and a suitable crack length, but the choice can only be determined by experiment. They are different for different materials and specimen geometry. The conditions are found by using an optimum crack-length to specimen-width ratio as recommended by Srawley and Brown [16] and then measuring K_c over a range of specimen widths. Because of the interference of the boundaries the apparent K_c increases with specimen width and the true K_c is the value reached at large strip widths.

Specimens with symmetric edge cracks were prepared as shown in Fig. 2. The widths were varied over the range $2b = 5$ to 10 cm, with $l/b = 3$ and $a/b = 0.35$ and over the range $2b = 15$ to 50 cm, with $l/b = 1$ and $a/b = 0.4$. The edge cracks were made with a sharp surgical blade. The specimens were securely clamped

between a pair of steel plates with Scotch brand double-coated tape spread on each side of the sheet, in an Instron Floor Model Universal Testing Instrument. This clamping arrangement appeared to provide uniform load distribution across the sheet and no sheet slippage was observed even up to the maximum load (2.5 kg cm^{-1}), encountered in this work. For each 10 cm specimen length, a cross-head speed of 1 cm min^{-1} was employed to load the specimen. A change in cross-head speed by a factor of 2 did not change the fracture load significantly as the crack propagation was rapid in each case.

The flawed paper sheet behaved like a brittle material, having a linear load-elongation curve that dropped rapidly to zero at failure. It was assumed, therefore, that no significant crack growth occurred before catastrophic failure. The initial crack length and the peak load determined from the load-elongation curve were used for calculation of K_c . The yield stress was determined separately from load-elongation curves of 3.8 cm wide strips, and was determined as the stress at which strain deviates by 0.2% from linearity [17]. K_c was determined from Equations 3 to 5, using the finite-width correction factor $F(a/b)$ computed by Bowie [18] for the specimen dimensions chosen. Results for σ_N/σ_{ys} and K_c as a function of specimen width are given in Figs. 3 and 4. The value σ_N/σ_{ys} falls below unity

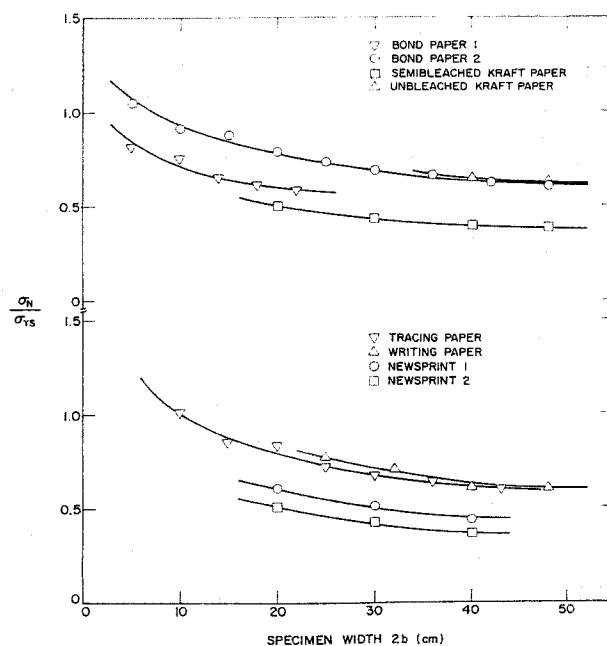


Figure 3 Effect of specimen width on the net tensile load at fracture. $a/b \approx 0.35$ to 0.4 . All points are averages of 7 to 10 tests.

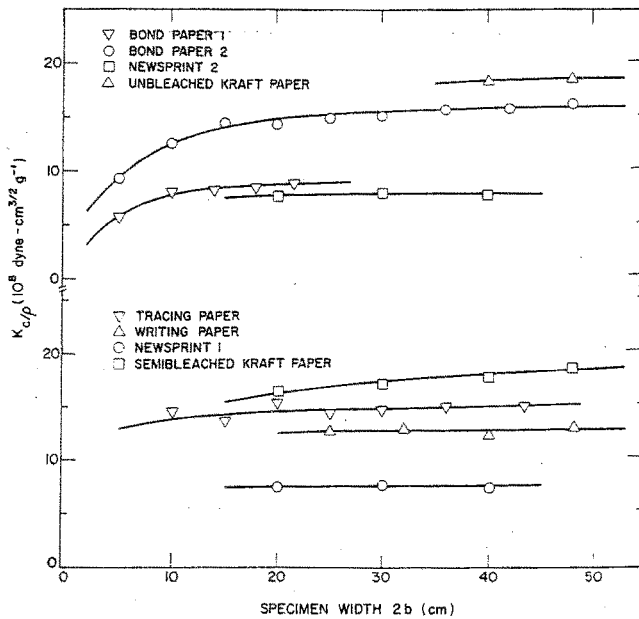


Figure 4 Variation of K_c/ρ with specimen width.

for all specimen widths above 10 cm. As expected, K_c increases asymptotically with increasing specimen width. However, the width at which K_c becomes sensibly constant evidently depends on the type of paper.

In order to determine G_0 , it is necessary to use Equation 6. The elastic constants a_{11} and a_{22} are respectively equal to $1/E_1$ and $1/E_2$ where E_1 and E_2 are the elastic moduli in the machine and the cross direction of the sheet. To determine $(2a_{12} + a_{66})$, use is made of the relationship [19],

$$\frac{1}{E_\theta} = \frac{\cos^4\theta}{E_1} + \frac{\sin^4\theta}{E_2} + (2a_{12} + a_{66}) \sin^2\theta \cos^2\theta, \quad (7)$$

where E_θ is the elastic modulus of the sheet at an angle θ measured from the machine direction. For $\theta = 45^\circ$, Equation 7 leads to

$$(2a_{12} + a_{66}) = \frac{4}{E_{45^\circ}} - \left(\frac{1}{E_1} + \frac{1}{E_2} \right). \quad (8)$$

The determination of elastic constants in Equation 6 reduces, therefore, to measuring three elastic moduli E_1 , E_2 and E_{45° .

3.4. Measurement of work of fracture R

As pointed out by Gurney and Hunt [8], the determination of the quasi-static energy of fracture in a material can be made by: (1) ensuring that the energy stored in the specimen and the testing machine at the moment of failure is small, so that catastrophic failure does not occur; (2) using crack and specimen dimen-

sions such that at failure the yield stress of the specimen is not exceeded in regions remote from the crack.

The following experimental conditions were found to meet these requirements. A hard testing machine (Instron Floor Model) was used. Tensile specimens with symmetric edge crack (Fig. 5) were clamped in two rigid line-type clamps capable of taking specimens as wide as 20 cm. The specimen length was maintained at 4 cm. Specimen width was 15 cm and a/b was maintained at 0.4. A cross-head speed of 0.1 cm min^{-1} was employed throughout.

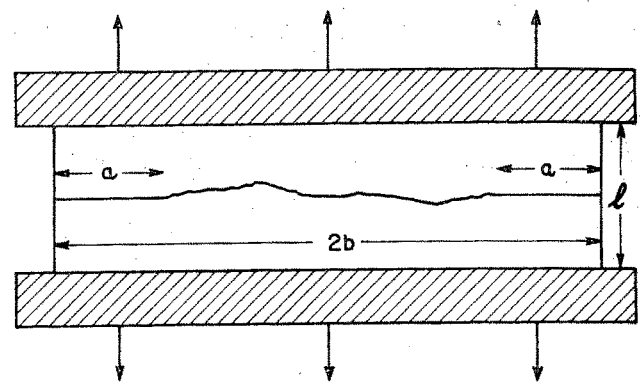


Figure 5 Geometry of specimen used for determination of R .

A typical load-elongation curve for quasi-static crack propagation in paper is shown in Fig. 6. The area under this curve, which is the work of

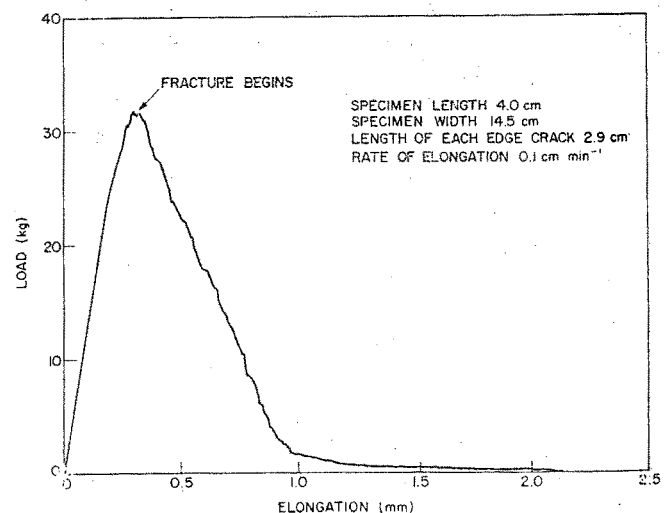


Figure 6 Load-elongation curve for a quasi-static fracture energy measurement on semibleached kraft paper.

fracture, was measured by an automatic integrator. The total fracture path was measured by a map reader. R was calculated as the work of fracture divided by the length of the fracture path.

4. Results and discussion

For a linear elastic material, the quasi-static fracture energy R and the critical strain energy rate G_c are theoretically equivalent [8]. Fig. 7 shows a plot of the experimentally determined values of G_c and R for eight different papers. The correspondence is excellent, in view of the assumptions made in the theories, and the experimental difficulties of meeting the theoretical requirements. This correspondence is strong support for the view that the methods of fracture mechanics can indeed be applied to paper.

A comparison can be made of our results with

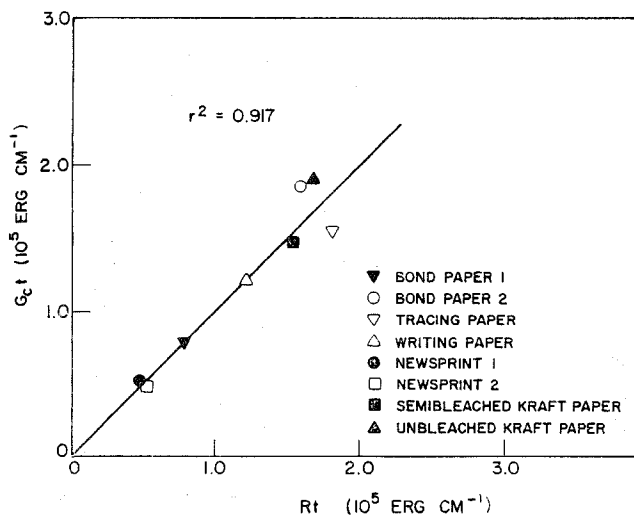


Figure 7 A plot of critical strain energy release rate ($G_c t$) against work of fracture (Rt). All points are averages of 10 tests.

earlier data [6, 7] on the fracture energy of paper. Balodis [6] measured the strain energy release rate G for various papers using the Griffith-Irwin relation of Equations 3 and 5, but without correcting for the plastic zone. He also measured a quantity G_σ , the strain energy release rate at constant stress using the compliance method [11]. For equivalent papers, his values for G are less than 10% and for G_σ , less than 30% of our results.

Andersson and Falk [7] also measured G for various papers using the Griffith-Irwin relation.

A correction, Δa , to the crack length, a , was made by fitting the experimental data to the equation $\sigma^2 = EG/\pi(a + \Delta a)$. Their estimates of the plastic zone size were large and this, in some cases, more than doubled the uncorrected G values. In spite of this large correction, their values for G are less than 20% of ours for equivalent papers.

The reason for the large discrepancy between these results and our own is clear. The material property G_c can only be measured accurately on samples sufficiently wide and crack lengths sufficiently long that the requirements given in Section 3.3 are met. For our papers, results that are erroneously low were obtained for specimen widths below 10 cm and the error increased rapidly with decreasing specimen width (Fig. 4). Both Balodis [6] and Andersson and Falk [7] used specimen widths of 1.5 cm, a value far too low for the measurement of fracture resistance. It must be concluded that Balodis and Andersson and Falk did not measure the material property G_c for paper, but some value lower than this, depending on the specimen size and other properties of the paper.

It is of particular interest to compare our values of G_c/ρ with those obtained on other materials. Table II shows such a comparison. It seems that on this basis paper ranks with some of the toughest materials such as steel and polycarbonate sheets, and certain fibre-reinforced epoxy composites.

The determination of R by quasi-static method is a simple one, and in a subsequent publication we intend to suggest a standard method that would be universally suitable for paper samples. The importance of the parameter R in the usage behaviour of paper is as yet uncertain but, because of its proven value in other fields, it seems worth investigation.

5. Conclusions

The fracture toughness, G_c , of paper can be measured using linear elastic fracture mechanics applied to a homogeneous orthotropic continuum, and taking care that all the conditions for the measurement are met. The quasi-static fracture energy, R , can also be measured. The two are experimentally equivalent, as suggested by theory. Compared to other materials on an equal weight basis, paper has a high fracture toughness.

TABLE II Typical values of G_c for various materials

	G_c (10^6 erg cm^{-2})	Approximate density ρ (g cm^{-3})	G_c/ρ (10^6 erg cm g^{-1})	References
<i>Metals</i>				
Aluminium-base alloys	7-105	2.7	2.6-39	a, b
Steels	0.6-900	7.9	0.08-114	b
Titanium-base alloys	10-40	4.5	2.2-8.9	b
Copper	100	8.9	11.2	c
Brass	60	8.4	7.1	c
<i>Polymers</i>				
Polystyrene (rubber modified)	8-9	1.07	7.5-8.4	c, d
Polymethylmethacrylate	0.9	1.19	0.76	c, d
Polycarbonate	6.7-54	1.2	5.6-45	b, d
Polyvinylchloride	66	1.4	47	d
Rubber	12	0.92	13	a
Cellulose acetate	2.5-5	1.3	1.9-3.8	a, c
<i>Woods</i>				
Douglas fir, parallel to grain	0.033	0.48	0.07	b
Douglas fir, normal to grain	0.228	0.48	0.48	b
Deal wood, normal to grain	4	0.5	8	c
Teak, normal to grain	12	0.8	15	c
<i>Composites</i>				
Graphite-epoxy, parallel to fibres	0.165	1.65	0.1	[23]
Graphite-epoxy, normal to fibres	20.5	1.65	12.4	[23]
Carbon fibre/epoxy, normal to fibres	5-85	1.65	3-51	[24]
Glass-reinforced plastics, normal to fibres	0.5-5	1.5	0.3-3.3	[25]
<i>Papers</i>				
Bond paper 1			10.5	This study
Bond paper 2			31.2	
Tracing paper			27.6	
Writing paper			21.4	
Newsprint 1			10.4	
Newsprint 2			10.1	
Semibleached kraft paper			30.2	
Unbleached kraft paper			42.8	

a Data tabulated in [11]

c Data tabulated in [21]

b Data tabulated in [20]

d Data tabulated in [22].

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