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TOPIC: STRUCTURAL TOPOLOGY OPTIMIZATION

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PLACE: 118 Nieuwland Science

ABSTRACT

Since the popular 1988 work of Bendsoe and Kikuchi, structural topology optimization has been a thriving research topic. In this talk I discuss the history of and three resolutions to the problem:

\[
\min_{f \in \Omega} \quad c_0(u) \quad \text{such that} \quad c_2(u, v) = 1 \forall v \in H_1^0
\]

where \( c_0 \) and \( c_2 \) are the energy bilinear and load linear forms associated with the equations of linear elasticity, \( u \) and \( v \) are test and trial displacement fields, \( c_0 \) is the cost function to be minimized, e.g. the compliance \( c_0 = l_0 \), \( f \) —the control to be optimized—defines the region \( \Omega \) over which \( c_1, c_2 \) and \( l_0 \) are defined and \( O \) is the set of admissible controls which provides smoothness and volume restrictions on \( \Omega \).

The genesis of topology optimization is attributed to the inability to generate holes in structural shape optimization wherein it is common to express \( \Omega \) as the image of a fixed region \( \tilde{\Omega} \) so that \( O = \{ f(\tilde{\Omega}) : f \text{ is a diffeomorphism} \} \), e.g. \( f \) can be a mapped mesh generator. The diffeomorphism restriction on the control \( f \) precludes the ability to introduce holes into the structure. This limitation is resolved in the topological derivative approach wherein an asymptotic analysis is used to determine the effect that a small hole of size \( \varepsilon \) placed at the location \( \varepsilon \in \Omega \) has on the value of the cost function \( c_0 \).

Another way to introduce holes is to define the set of admissible controls as \( O = \{ \chi : \chi(x) = 0 \text{ or } 1 \text{ for all } x \in \Omega \} \) where \( \chi \) is the characteristic function of \( \Omega \) and \( \Omega_L \) is the fixed hold-all. This leads to an ill-posed problem which can be physically justified by noting that a series of many strategically placed fine holes renders a stiffer structure than a series of fewer strategically placed large holes of equal net volume. The limit in this refinement procedure, and hence the solution to the topology optimization problem does not exist. To make the problem well-posed the admissible control set \( O \) is changed in one of two ways. In the relaxation method generalized designs are introduced via perforated composite materials with infinitesimal holes so that \( O = \{ f : 0 \leq \rho(x) \leq 1 \text{ for all } x \in \Omega_L \} \) where the control \( \rho \) is the solid material volume fraction. This approach requires homogenization to define the elasticity tensor and the allusive \( G \)-closure to quantify the range of elasticity tensors for a given volume fraction \( \rho(x) \). In the restriction method constraints are placed on \( \chi \) to provide a minimum length scale for the geometric features of \( \Omega \), e.g. by limiting the perimeter.

The aforementioned methods are amenable to computation and we provide the following linear elastic examples to illustrate them:

1. To compare the methods, the classical problem of minimizing the compliance of a cantilever beam subject to a volume constraint is solved via all three methods.
2. To illustrate the ability to accommodate localized constraints, the minimal weight design of a bracket subject to stress constraints is solved via restriction.
3. To demonstrate the ability to accommodate dynamics, maximal energy focus and dispersion designs of two-phase composite plates subject to impact loads are solved via relaxation, and
4. To motivate other applications of the topological derivative, energy release rates are estimated for cracks of any small size, at any boundary location and at any orientation from a single finite element analysis on a coarse mesh.

NOTE: If you are interested in meeting individually with Prof. Tortorelli, please contact Evelyn at 631-5431